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Title Firm-oriented policies, tax cheating and perverse outcomes

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Firm-oriented policies, tax cheating and perverse outcomes*

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Abstract

This paper examines the implications of firm-oriented fiscal policies, namely investment subsidies and tax allowances, in an economy where producers may potentially avoid taxes. Among our results we stress the following. First, although investment subsidies induce increased capital accumulation (a level effect), they promote tax evasion; these subsidies induce firms to increase actual capital accumulation (a level effect), but also produce a reduction in the share of aggregate capital stock deployed in taxed, "official" production (a composition effect). **Second**, parameters characterizing the tax-enforcement system play a major role in explaining tax evasion and firm size. Third, the technology structure matters for determining how to allocate resources between official and unofficial production.

Keywords: State aid, tax exemptions, investment subsidies, tax evasion, unofficial / underground production, investment.

JEL classification: E26, E22, H25, H26.

1 Introduction

This paper examines the impact of fiscal policies aimed to support firms, in particular investment subsidies and tax allowances, on tax evasion and underground production.

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Subsidy programmes and tax advantages for "infant" industries or depressed areas are often justified because the industry is not competitive enough and prices do not show full flexibility. Granting capital subsidies and tax allowances to firms also has important implications for underground activities and tax evasion.

Underground activities occur in most countries, and there are significant indications that this phenomenon is widespread and increasing. The estimated average size of the underground sector, as a percentage of total GDP, in the late 1990s was about 17 percent in OECD countries (Schneider and Enste, 2002).[1]

The paper presents an optimal-investment model in which a representative firm maximizes its expected cash flow, choosing simultaneously an optimal combination of aggregate capital stock (i.e. firm size) and its allocation between official and unofficial production, conditional on a set of fiscal policy and technological parameters.[2]

We are not aware of any contribution investigating how firm-oriented fiscal policy affects optimal investment in official vs unofficial production. This is a major issue, because underground activities represent an additional financing source for investment, which is not subject to distortionary taxation. This means that fiscal policy results might differ from what we expect in a model that explicitly incorporates tax evasion.

The model presented in this paper focuses on the moonlighting firm, by which we mean a firm that operates simultaneously in the official and unofficial sectors, using the same stock of capital while evading taxation for the activities that are undertaken underground. Such a firm is able to evade taxation, like firms that operate only in the underground economy, but in addition, it can exploit technological/institutional advantages conferred by its above ground activities.

Our analysis focuses on the Italian economy due to the sizeable underground sector in Italy, and the high percentage of "moonlighters" which operate in the unofficial sector.[3] Furthermore, Italian governments have repeatedly supported firms with investment subsidies and tax allowances. We think that the theoretical scheme and its predictions may be applied, without loss of generality, to other countries.

Our analysis delivers three main results: First, investment subsidies promote tax evasion; subsidies induce firms to increase their capital stocks (a level effect), but also produce a reduction in the share of capital used in official activity (a composition effect). An investment subsidy policy is a non-excludable public good that opens room for free-riding (tax evasion in other words). In this context the Government is not capable, because of (un-modelled) monitoring costs, to distinguish between regular and moonlighting firms. Firms therefore have an incentive to declare a sufficiently small amount of revenue to be eligible for the subsidy, while investing relatively more in the underground economy and "pocketing the tax wedge". Second, the tax-enforcement system plays a major role in determining both the firm's size, affecting aggregate capital stock, and tax evasion. The third main result of our analysis asserts that technology matters (what we call moonlighting effect and Total Factor Productivity effect, discussed below) for determining how to allocate resources between official and unofficial production, hence the amount of reported revenues.

The structure of the paper is as follows. Section 2 provides some stylized facts and elaborates the motivation of the paper. Section 3 explains the firm's maximization problem and characterizes the long-run equilibrium. In Section 4 the main results of technology and

Table 1: Italian firms by employment and revenues size, 2002

No. of Employees	No. of firms with Revenues:		
	below Euro 5.16 million	above Euro 5.16 million	Total Firms
1-9	4,379,107	-	4,379,107
10-19	120,627	7,216	127,843
20-49	33,152	18,521	51,673
above 50	-	22,924	22,924
Total Firms	4,532,887	48,660	4,581,547

policy analysis are reported and discussed. Finally, in Section 5 we make some concluding remarks.

2 Fiscal Policies and Tax Evasion: Selected Stylized Facts for Italy

The empirical evidence for Italy shows that the production structure, tax evasion and industrial policies are intertwined. In Italy a large proportion of firms are very small, as shown by fiscal data (Revenue Agency) reported in Table 1.

In 2002, the amount of Euro 5.16 million of sales represented the threshold, adopted by the Revenue Agency, to distinguish small and medium-sized firms (hereinafter SMEs), which are compelled to file their tax reports using a procedure called Studi di Settore (Santoro, 2008). In the period 1999-2005 the vast majority of Italian firms (about 99%) were classified as SMEs, and employed fewer than 10 workers. The average size of declared revenues was considerably below the threshold, amounting to Euro 200,000 in the period considered, which allowed most of the firms to adopt a simplified bookkeeping system (Convenevole, Perinetti, 2008). This raises critical issues in terms of tax evasion, since a recent survey (Di Nicola and Santoro, 2000) pointed out that tax evasion is very widespread among small firms and new firms, especially those located in the south of Italy. Evidence provided by Ercoli (2005) suggests the occurrence of a threshold effect, driving firms to stay small in order to remain under the legal limit granting the simplified bookkeeping system. The latter indirectly enhances the opportunity of successful tax evasion, since it reduces the effectiveness of fiscal controls. Ercoli's data also suggest that small firms are mostly involved in tax evasion through under reporting of revenues, whereas large firms mainly evade taxes by inflating costs. The share of tax evasion in the sample used by Ercoli (2005) is displayed in **Figure 1**, confirming that evasion declines with the size of apparent revenues.

Figure 1 also shows that there is large heterogeneity, especially among the smallest firms, where evasion ranges between 5 and 45% of apparent revenues; however, most of the firms display evasion under 15% of total revenues.[4]

Concerning the structure of tax evading enterprises, Censis (2005) confirms that most of them are only partial tax evaders, while firms completely unknown to fiscal authorities are a residual share, as it is is shown in **Table 2**.[5]

Given the external appearance of regularity, nothing prevents these firms from competing

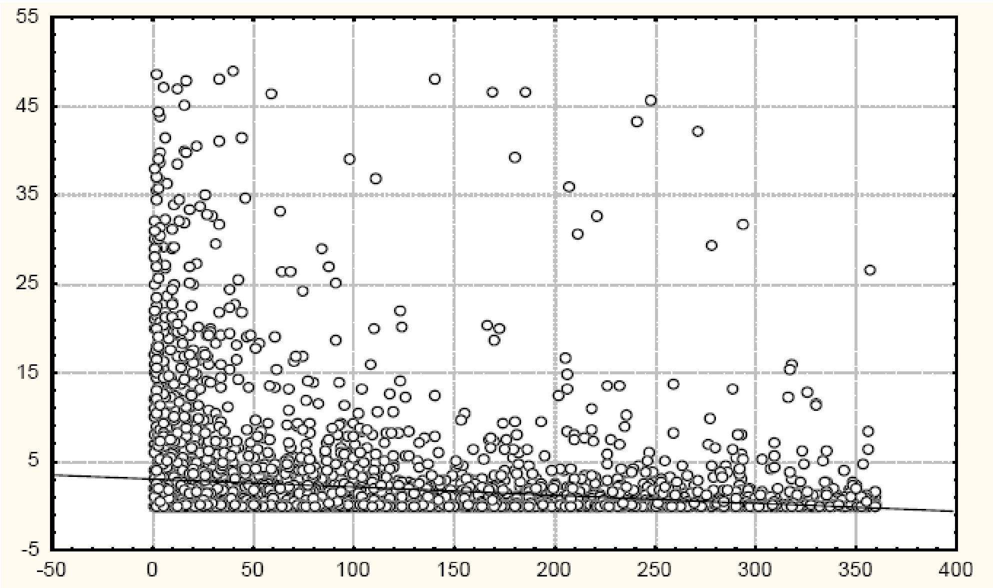

Figure 1: Tax evasion (as a share of total revenues, vertical axis) and total revenues (thousands of Euro, horizontal axis) based on fiscal audits. Source: Ercoli (2005)

Table 2: Irregular firms as a percentage of total firms, main Italian regions, 2005

Irregular Firms	North-West	North-East	Center	South	Italy
Full tax evasion (1)	5.8	4.9	6.8	16.8	9.7
Under reporting (2)	29.7	31.3	41.7	59.9	43.4
Total Irregular Firms (1+2)	35.5	36.2	48.5	76.7	53.1

Source: Authors' calculation from Censis (2005).

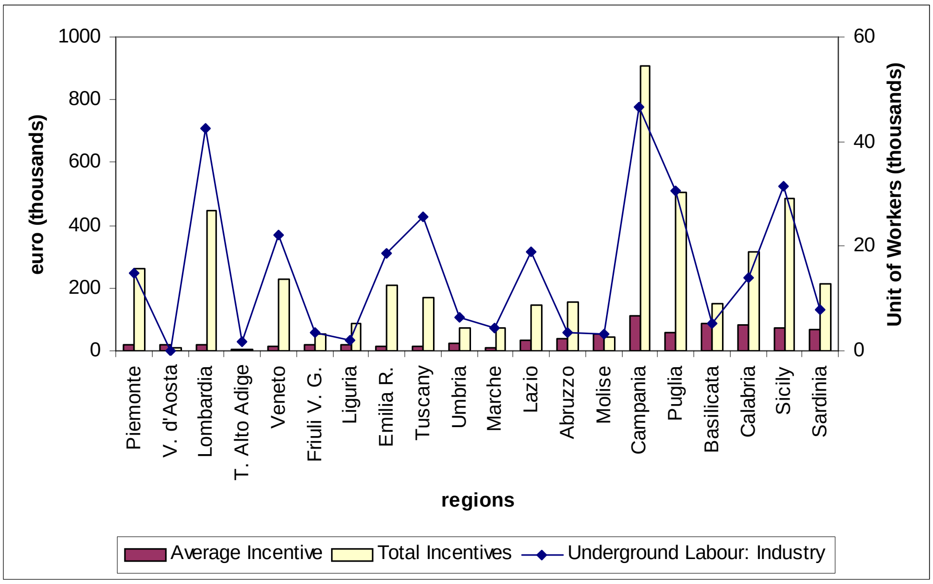

Figure 2: Average incentives (red bar, left side axis); total incentives (yellow bar, left side axis); irregular employment (blue line, right side axis) in Italian regions. Source: Ministero Attività Produttive (2005) and Istat (2003).

for fiscal benefits, such as tax relief policies for investment expenditure or tax allowances. Actually, evidence about State Aid policies, in Italy as well as in Europe, shows that most public funding is directed to SMEs.[6]

While the justification for State aid policies has often been discussed, their implications in contexts characterized by the presence of the unofficial economy are largely neglected. However, the above-described evidence plausibly envisages interactions between policies to support firms and tax evasion. **Figure 2** reports investment incentives and numbers of irregular workers for each of the 20 Italian regions. Casual inspection suggests that there is a positive correlation between the two measures, which is particularly marked when considering irregular workers in the industrial sector (the correlation is 0.88).

The most intuitive explanation of the positive correlation between the two indexes is typically traced back to the common factor of "underdevelopment" (Loayza, 1996; Johnson et al., 1997). However, in this paper we introduce an additional argument, that is a dishonest behaviour of firms operating in areas with large fiscal benefits and a large underground economy, as shown in the following sections.

3 The Model

3.1 Tax Evasion

Empirical evidence suggests that firms have three environments to cope with. There exist (possibly large) firms operating in competitive markets with an efficient technology that are not involved in "underground" activities, whereas less efficient firms face difficulty in competing without under-reporting (moonlighting) or concealing all their revenues (ghost firms).[7]

Participation in the official economy confers benefits from several services supplied both from government and from the "private" market; however, engagement in the official economy requires tax liabilities. On the other hand, tax evasion is a risky activity and, at least for the wholly underground ghost firms, it prevents them from gaining access to benefits available in the official economy.

If we define total output as Y and officially (unofficially) produced output as Y^R (Y^U), for a given triplet of tax-enforcement parameters (τ, ρ, s), i.e., a proportional income tax rate, τ, a fixed probability of being detected and fined, ρ, and a fixed surcharge $s > 1$ for income (discovered by the authorities) to be concealed, then the expected after-tax production is:

$$(1-\tau)Y + \tau(1-\rho s)Y^U, \tag{1}$$

where the second term in **Eq. 1** stands for the expected return of tax evasion, and is given by:

$$\begin{cases} \tau & \text{with probability} \quad (1-\rho) \\ \tau - \tau s & \text{with probability} \quad \rho \qquad s > 1 \end{cases}$$

In this economy the two extreme situations of full compliance and total evasion occur, respectively, when $\tau = 0$ and, in the case of confiscatory taxation, $\tau = 1$. In the former case, no positive returns arise from tax evasion, as it is clear from **Eq. 1**, whereas in the latter situation no positive after-tax returns occur in official production.[8]

More realistically, we are interested in studying the implications of tax evasion and assume that Condition 1 is fulfilled in the remainder of the paper:

Condition 1 $0 < \tau < 1$; $(1 - \rho s) > 0$.

This condition ensures that firms would have an incentive to produce both officially and unofficially.[9]

3.2 The Moonlighting production structure

Tax evasion can be considered a useful strategy to squeeze costs, particularly in the case of inefficient firms that might otherwise be pushed out of the market. It is often assumed that firms operating in the underground economy are less efficient than regular ones;[10] although this is plausible, we argue that firms operating simultaneously in the official and unofficial sectors might overcome some of the shortcomings of fully underground production.

The empirical evidence previously presented suggests that there could be some advantages for inefficient firms to stay partly in the official economy, and paying low taxes, rather than being completely ghost.

In our setting, the firm optimal size, K, is determined by understating the true value of the tax base (profits), by allocating only a share of total capital, μ, to honestly stated output. The key element here, compared to the literature on ghost firms, is that all services available on the official markets produce a flow of revenues which, once obtained by the "visible" side of the firm, would be freely available also to support its unofficial activities. Through appropriate under-reporting, the firm is able to benefit from participation in the official market: access to credit, banking services; public sector services, such as loans and capital subsidies; marketing and advertising services; trade marks, and so on.

If we assume, as is plausible, that most of the services arising from the official market participation are proportional to firm size, we can model an external effect generated by underreporting, and not exploitable by wholly underground, "ghost" firms, which we call the *moonlighting effect*. The latter, in a simplified framework with a single input of production, can be modelled as a total capital externality, allowing Total Factor Productivity (TFP) to be endogenous:

$$
\begin{aligned}
Y_{MOONL} &= B\left(\mu K\right)^{a} + B'\left[\left(1-\mu\right)K\right]^{a} \\
B' &= K^{a\sigma}.
\end{aligned}
$$

In the above production function, a is the capital elasticity, B is the exogenous TFP, and B' is the endogenous TFP. The quantity of total capital enhances the productivity of unofficial production; the moonlighting effect acts as trading inside two plants of the same firm. Choosing to allocate a share μ of total capital to the official economy implies a cost in terms of taxation, but involves a benefit in terms of an external effect going from the official to the unofficial production, i.e. the allocation of capital to the official production, $\mu > 0$, is an externality-producing activity.[11]

This technology can capture different models of unofficial production: medium-sized productive units, largely regular, with their own brand, which exploit underground production to gain extra profits; partial decentralization by a regular firm toward smaller and irregular productive units referred to as "local underground districts"; small firms producing largely underground, which use official production as a convenient screen to avoid fiscal controls.[12]

Condition 2 below suggests that the size of the externality should be sufficiently low as to ensure that returns to scale are not increasing at the firm level:

Condition 2 $0 < \sigma < \frac{1-a}{a}$.

The restriction on the size of the moonlighting effect σ and, consequently, the exclusion of any sort of increasing returns of scale, is a necessary assumption to allow the moonlighting firm to choose a finite optimal scale for its capital stock, capturing the positive interaction between official and underground production.[13]

When the firm engages in moonlighting activity, it exchanges a share of the exogenous TFP (B) for an endogenous one ($K^{a\sigma}$).[14] Moreover, for a given stock of capital, the entrepreneur compares expected profits under total and partial tax evasion, respectively:

$$
\begin{aligned}
&(1-\rho\tau s)\left(BK^{a}-rK\right); \\
&(1-\tau)\left[(\mu K)^{a}-rK\right]+(1-\rho\tau s)\,K^{a\sigma}\,(1-\mu)^{a}\,K^{a}
\end{aligned}
$$

and takes into account that only when using moonlighting technology can he/she apply for the tax benefit provided when investing, α. The optimal capital allocation involves a complex scenario of technological (B, a, σ) as well institutional (ρ, τ, s) parameters.[15] We show, in the FOCs in the next sections, how they influence the firm's capital allocation among the two productions.[16]

3.3 Value of the Firm

At time zero the firm is endowed with a given positive amount of capital $\left(\bar{K}_0\right)$, and with an intertemporally fixed flow of a non-capital resource (labour, land), which are normalized to unity.

Each instant a firm maximizes the intertemporal cash-flow function, choosing how many resources to allocate to official production, μ, and how much revenue to invest, I. Investing is a costly process for firms; the standard assumption here adopted is that the adjustment costs are a convex function of the rate of change of the capital stock:

$$C(I) = I^b; b > 1.$$

In addition, we assume that investments are encouraged by the government, which provides a capital contribution proportional to total investment, α, to firms which are willing to increase their capital stock. We assume that government is neither able to know whether new capital will be employed in official or unofficial production, nor has accountability tools at its disposal enforcing the firm to declare only the capital officially employed.[17]

The value of the firm is the expected present value of its revenues net of expenditures on capital input and costs incurred by adjusting its capital input. The representative firm maximizes expected cash flow $\mathcal{V}$ subject to a constraint set:

$$\max_{\{I,\mu\}} \mathcal{V} = \int_{t=0}^{\infty} e^{-rt}\Pi dt \tag{2}$$

$$s.to : \Pi = (1-\tau)\left[B(\mu K)^a - rK\right] + (1-\rho\tau s)(1-\mu)^a K^{a(1+\sigma)} - I - I^b + \alpha I \tag{3}$$

$$: \dot{K} = I - \delta K \tag{4}$$

$$: 0 \leq \mu \leq 1 \tag{5}$$

$$: \bar{K}_0 > 0 \tag{6}$$

$$: \lim_{t\to\infty} e^{-rt}\phi_0 K = 0 \tag{7}$$

$$: \alpha \in (0,1)\,; \rho s < 1; 0 < \tau < 1; 0 < \sigma < \frac{1-a}{a}. \tag{8}$$

The quantity $(1-\tau)\left[B(\mu K)^a - rK\right] + (1-\rho\tau s)(1-\mu)^a K^{a(1+\sigma)}$ represents the firm's expected revenues, net of taxation, I is the amount of gross investment, and δ is the physical depreciation rate of capital. The amount αI denotes an investment allowance, where α falls in the $(0,1)$ interval. Alpha represents several different types of State aid, such as grants to firms investing in less developed areas (regional aid), loans to small and medium-sized enterprises (SME aid), and other facilities for specific sectors (sector aid).

Defining ϕ_0, ϕ_1 and ϕ_2 Lagrange multipliers and the current value Hamiltonian $\mathcal{H}$, manipulation of the first order conditions leads to the following conditions characterizing optimal capital accumulation and tax evasion:[18]

$$\left[(\phi_0 - 1 + \alpha)/b\right]^{1/(b-1)} = I \tag{9}$$

$$(1-\tau)aB\mu^{a-1}K^a - (1-\rho\tau s)a(1-\mu)^{a-1}K^{a(1+\sigma)} = 0 \tag{10}$$

$$\dot{\phi}_0 = (r+\delta)\phi_0 - (1-\tau)\left[aB\mu^a K^{a-1} - r\right] - (1-\rho\tau s)(1-\mu)^a a(1+\sigma)K^{a(1+\sigma)-1} \tag{11}$$

$$\dot{K} = I - \delta K. \tag{12}$$

Proposition 1 below proves that the model has an interior solution.

Proposition 1 *For firms with moonlighting technology it is not worth either becoming completely regular* ($\mu = 1$) *nor turning into a ghost firm* ($\mu = 0$)*: i.e. the model does not admit corner solutions.*

Proof. μ as well as $(1-\mu)$ are the basis of a negative power in **Eq. 10**, such that to have a finite solution they must necessarily lie in the open interval $(0,1)$. ■

The investment function (**Eq. 9**) has standard characteristics: for a given level of fiscal allowances, α, investment is increasing in ϕ_0, and gross investment is zero when the marginal value of capital is just equal to the market price of capital, normalized to 1, net of fiscal allowance. Fiscal incentives for capital accumulation clearly increase investment.

Eq. 10 ensures the optimal allocation of capital between visible (official) and underground (unofficial) production: the marginal effect of a capital reallocation on the net-of-tax revenues in the two sectors must be equal.

3.4 The Steady State

3.4.1 Qualitative Analysis

Combining the investment function with **Eq. 12**, we obtain a dynamic system such that the Steady state ($\dot{\phi}_0 = \dot{K} = 0$) is characterized by the system:

$$\begin{cases} \phi_0 = \frac{1}{(r+\delta)} \left[(1-\tau) \left(aB\mu^a K^{a-1} - r \right) + (1-\rho\tau s)(1-\mu)^a a(1+\sigma) K^{a(1+\sigma)-1} \right] \\ I(\phi_0) = \delta K \\ \mu(K) = \dfrac{K^{\frac{a\sigma}{a-1}} \left(\frac{(1-\rho\tau s)a}{(1-\tau)aB} \right)^{\frac{1}{a-1}}}{1 + K^{\frac{a\sigma}{a-1}} \left(\frac{(1-\rho\tau s)a}{(1-\tau)aB} \right)^{\frac{1}{a-1}}} \end{cases} \tag{13}$$

The first equation suggests that in equilibrium (i.e. in the long run) the shadow price of capital is the discounted value of the net-of-tax marginal productivity of capital; the second condition states that the stock of capital is stable when investment is just equal to physical depreciation of capital; finally, the last relation expresses the optimal allocation of the capital stock between official and unofficial production.[19] It should be stressed that the long-run equilibrium can only be described in a three-dimension space, and given the non-linearity of the involved relationships, we are compelled to use calibrate and simulate the system in order to describe the nature of the steady state.

Proposition 2 *In the long run, the dynamic system pertaining to steady state System 13 admits a unique steady state.*
Proof. *APPENDIX* ■

Proposition 3 *The steady state of the dynamic system pertaining to steady state System 13 is always a saddle path.*
Proof. *APPENDIX* ■

3.4.2 Model parameterization

The model depends on five parameters. We calibrate these parameters for Italy, a country with a large-scale underground production and high tax evasion.[20] Moreover, as stressed earlier in this paper, underground activities are characterized by a high percentage of moonlighting firms – firms that simultaneously produce officially and unofficially – and that Governments in this political economy have repeatedly supported firms with capital subsidies and tax allowances.[21]

The capital elasticity a, consistent with the calculations of capital's income share from the national accounts, is set at the value 0.3; the exogenous cost of capital (the real interest rate), r, set at 0.025; the rate of physical depreciation of capital, δ, is calibrated to 0.125. Next, the tax rate, τ, is set at 0.4, which is the average between the IRES (Imposta redditi società, a sort of profit taxation) and the effective average taxation, as calculated by Chiarini et al. (2008); the tax surcharge applied to firms caught evading taxes, s, following Italian civil law, is set at 1.3. The probability that a firm is discovered and convicted of tax evasion, ρ, is set at a very low value, 0.05, to give an idea of low enforcement, which intuitively corresponds to actual conditions in Italy. This is particularly true for small firms, since, according to Ercoli (2005), most of the inspections are directed towards large firms. The size of the subsidy for capital accumulation, α, is set at 14% in the baseline calibration.[22]

Finally, we need to specify the two technological parameters B and σ. These are jointly calibrated to get an average evasion plausibly consistent with data available for Italy. The moonlighting effect must also be consistent with Condition 2 defined above, and with the model saddle path's condition, outlined in the Appendix. As the maximum value for this parameter is 1.6, we set it at an intermediate level of 1, such that the total capital externality is 0.3. Finally, given data on tax evasion noted in Section 2, a plausible figure for tax evasion is 20%; accordingly, we set the exogenous Total Factor Productivity, B, to a value of 10.[23]

Model Calibration: the benchmark

α	B	τ	σ	r	ρ	s	a	δ
0.14	10	0.40	1	0.025	0.05	1.3	0.3	0.125

Given this set of parameters, the solution of the dynamic system identifies a single long-run equilibrium, given by the equilibrium vector:

$$(K^* = 19; \mu^* = 0.79),$$

with a benchmark capital size equal to 19, and a share of aggregate capital deployed in the official production amounting to 79%.

3.4.3 Steady State Relations (K, ϕ_0) and (μ, K)

The three steady state relations expressed by **System 13** can be geometrically represented in the space (K, ϕ_0, μ). In order to provide more insights into the local dynamics around the steady state, we prefer to represent them in two bi-dimensional graphs as in **Figure 3**.[24] The left-hand panel in the Figure displays the two steady state relations $\dot{\phi}_0 = 0$ (the shadow price of capital) and $\dot{K} = 0$ (the stock of capital), which are standard in the literature on optimal investment.

The right-hand panel in **Figure 3** represents, in the space (μ, K), the relationship between the official capital share μ and the total stock of capital (K), defined by the last equation of **System 13**: for each level of K identified by the solution of **System 13**, a unique cash-flow maximizing value of μ is identified. The locus $\mu(K)$ is monotone and decreasing: given the nature of the moonlighting effect, the larger the amount of total capital, the more benefit is obtained in shifting it to underground production (e.g. μ drops).

The left-hand panel in **Figure 3** also displays the local dynamics: the stability arrows show that there is a single stable arm which leads the firm toward the long-run equilibrium, consistent with the standard literature. When the capital stock dimension is lower than the optimal level, given **Eq. 10**, the official capital share, μ, is higher than optimal (see also the right-hand panel in **Figure 3**); during the process of capital accumulation, the firm also shifts capital into underground technology (i.e. μ drops). This allocating process lasts until the marginal productivity is equal across sectors (official and unofficial sectors, see. **Eq. 10**). An analogous symmetric process applies when the capital dimension is higher than the optimal level and the firm operates on the lower and right-hand side of the stable arm.[25]

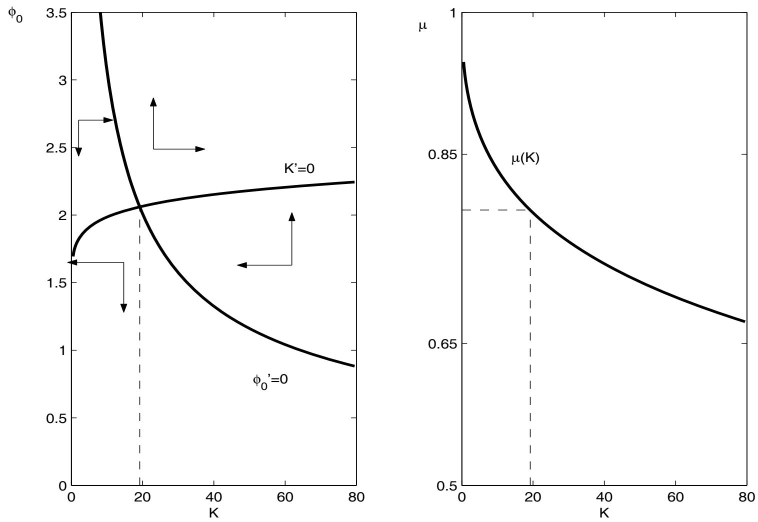

Figure 3: Points above the locus $\dot{\phi}_0 = 0$ are characterized, for each level of K, by a ϕ_0 higher than the equilibrium level; given the dynamic expressed in Eq. 11 it implies a growth in the shadow price of capital (arrows pointing up). Similarly, when considering points above $\dot{K} = 0$, we register for each K a ϕ_0 higher than the equilibrium level; given the investment function, Eq. 9, and the dynamic expressed in Eq. 12, it implies a growth of capital stock (arrows pointing right).

3.4.4 The Exogenous TFP effect

Parameter B is the exogenous component of the Total Factor Productivity (TFP) in moonlighting technology, and it produces important implications for the relationship between firm size and underground activity.

Changes in this technological parameter generate considerable differences with the baseline calibration, both in terms of optimal capital dimension and official capital share. In **Figure 4** we show the pattern of steady state capital size and its official allocation, calculated by solving System 13, for a set of possible parametrizations of the exogenous TFP. Precisely, we get the pattern of the steady state capital size (as well as μ) changing B in the interval $(0, 100)$, keeping unaltered all the remaining parameters at their benchmark.

As long as the exogenous TFP rises, there is a corresponding increase in capital stock, i.e. firm size. On the other hand, a larger TFP also implies that capital is increasingly allocated in the honestly stated production, i.e. μ rises, and the pattern displayed in the right-hand panel of **Figure 4** shows a convex relationship, slowly approaching unity as TFP tends to become very large.

These experiments suggest a strong and direct relationship between the exogenous TFP, which predicts the degree of efficiency of the technology and drives firm size, and the choice to operate regularly. This is consistent with the evidence commented upon in the previous section, and with the literature, that is almost unanimous on the importance of firm size in

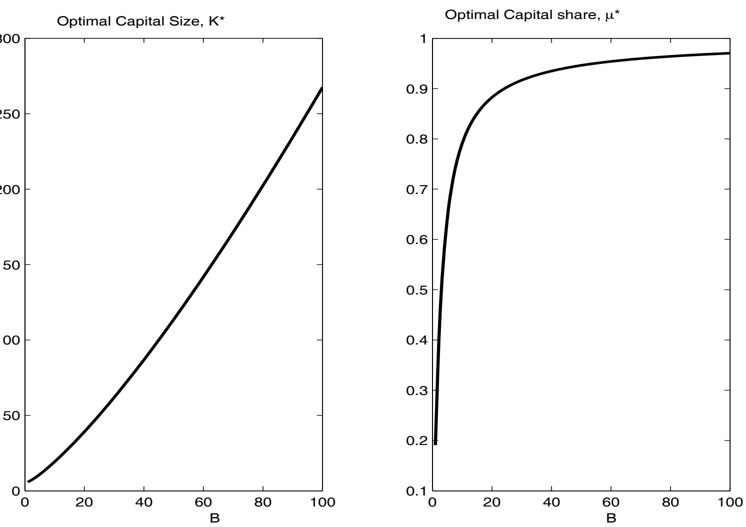

Figure 4: The effect of exogenous TFP (B) on optimal capital stock (K*) and official capital share (μ*).

affecting the propensity to operate in the underground economy.

Two further points are worth emphasizing before proceeding. First, the model states that for small and poorly efficient firms it is worth declaring some of their revenues, i.e. μ is always above zero, but the visible production is considerably lower than total production. Secondly, as long as TFP and firm size rise, the share of capital allocated to official production tend to one: a process of technological growth should be able to promote compliance.

4 Policy Implications

This section presents the effects of selected fiscal policy experiments, pertaining to the enforcement triplet (τ, ρ, s) and to the subsidy policy (α), on the long-run equilibrium of the moonlighting firm (the total capital stock, the officially deployed capital share, and total production). Moreover, we consider how different values for the technological parameter σ affect the size of the moonlighting firm, and the allocation of the total capital stock between the two productions. Notice that parameter σ can also be broadly considered a policy instrument, in the sense that the possibility for the moonlighting firm to exploit the external effect of the aggregate capital is supposed to be a function of the institutional and social framework in which firms operate.

Useful insights may be drawn from the analysis of the reaction functions, which express the estimated steady state of total capital, its officially deployed share and total production as a function of each single fiscal policy parameter. Each graph in the top row of **Figure 5** shows, respectively, the effects of a variation in tax rate, capital subsidy, expected penalty and moonlighting effect on the size of total capital stock, while in the middle and bottom rows the reaction functions for the officially deployed capital capital, (μ) and total production are displayed.

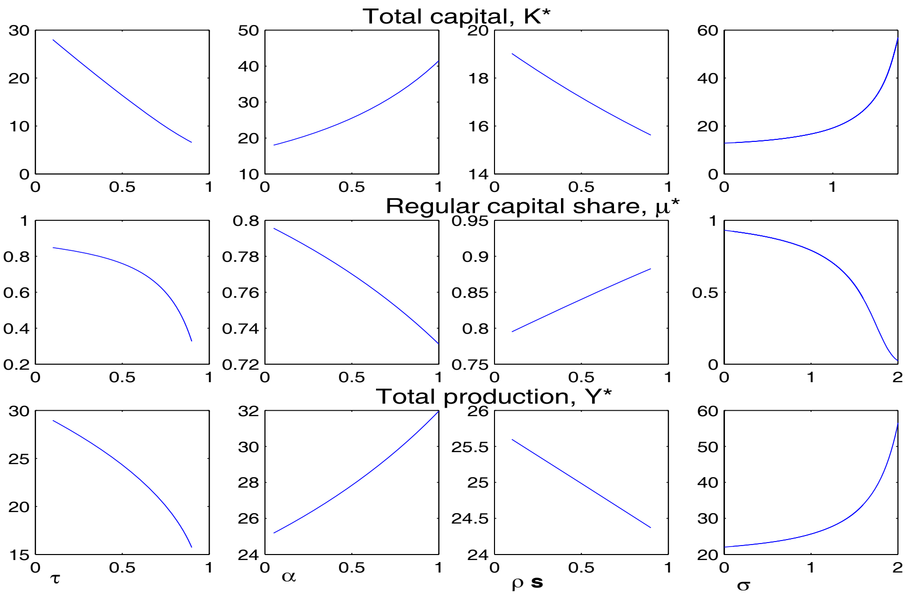

Figure 5: reaction of the equilibrium capital stock (K*, top row), equilibrium capital allocation (μ^*,middle row) and total production (Y^*, bottom row) to: taxation (column 1); investment subsidy incentives (column 2); expected penalties for discovered tax evasion (column 3); moonlighting effect(column 4).

The reaction functions are always monotone, though non-linear; we comment upon each of the fiscal experiments separately in the next subsection.

4.1 Sensitivity Analysis

Starting from the reaction functions to changes in taxation, **Figure 5** shows that tax policy is the only measure able to generate a positive co-movement between the two objective variables, total capital and official capital share. For instance, starting from the baseline value of taxation, 0.4, a fall in tax rate generates increases in the total capital stock as well as its officially deployed share. The tax reduction increases the net-of-tax marginal revenue of capital; this occurs in more marked fashion in officially produced output due to **Condition 1**.[26] The fall in the taxation ratio, $(1-\rho\tau s)/(1-\tau)$, alters the equilibrium relationship between K and μ expressed in **Eq. 10**. Hence, a tax cut (rise), *ceteris paribus*, induces the moonlighting firm to engage in more (less) official production.

When considering the second column, i.e. the effect of investment subsidies, the following remarks apply. Increasing the size of subsidies to capital accumulation pushes up the equilibrium level of the capital stock, as we would intuitively expect, but also generates a marginal reduction in the official capital share. The increase in the incentive to capital accumulation reduces the cost of capital, such that there is an immediate effect on investment. The rise of capital stock alters equilibrium marginal productivity (**Eq. 10**), such that as long as net investment is positive, the firm also reallocates capital between the official and unofficial sectors. The important point is that, contrary to the presumption that subsidies may also be useful for pushing firms to operate "overground", in the presence of moonlighting technology, the incentives to improve capital stock are actually counterproductive in that they increase the unofficial economy overall.

From the third column of **Figure 5** it is possible to infer the impact of a stricter enforcement policy, as simulated by a larger value for the product (ρs), i.e. the expected penalty for discovered underground production. The graph clearly comfirms that increasing the effectiveness of enforcement causes a decline in the share of capital allocated to unofficial production. However, there is also a negative effect on total capital, as well as total production. This evidence, associated to the strong and positive effects of a tax cut, seriously depreciates the role of enforcement in fighting tax evasion.

Finally, we examine the impact of the external effect, σ. This new technological element characterizes our firms, explicitely stating the advantages of operating in the official and unofficial productions. The moonlighting effect, proposed in this paper, shows how the evader and the regular entrepreneur are "intertwined", and how great is the advantage of operating together through the economy as a result of transparent support of official activity for unofficial production. This insight follows directly from the model and, as stressed above, it is supported by empirical evidence, emerging from several surveys, on the characteristics of firms that operate in the underground sector. Intuitively, a large (small) value for σ implies that the moonlighting firm strongly (weakly) benefits from the simultaneity of its two productions. For instance, a larger value of σ compared to the benchmark ($\sigma = 1$) triggers an investment process and a drop in the officially deployed share of capital. While σ approaches the threshold excluding increasing returns of scale, the solution converges toward a ghost firm, i.e. $\mu \to 0$, while size is no longer determinate. [27]

Two main policy implications are worth stressing when focusing on policies against tax evasion. First, a trade-off arises when a stricter penalty is enforced: in this situation, as shown clearly by the third column of **Figure 5**, the rise in official capital is associated to a decline in total production. Secondly, tax evasion, as proxied by the official share μ, seems to display a hard core which is, for plausible fiscal policy parameters, around 20%. This proportion is hardly affected either by a rise in expected penalty and/or by larger capital subsidies. In the first case (plots in column 3), the capital allocation to the apparent/official production ranges from .78 to .88 while the expected penalty rises; in the second case (column 4), a perverse effect arises.

Conversely, tax evasion is appreciably affected, under the moonlighting technology, by the size of the externality effect, σ, and tax rate, τ. Moreover, though a large externality implies an officially deployed share of capital tending to zero, a considerable reduction of the fiscal burden is unable to allocate all the capital to honestly stated activities.

On the other hand, in evaluating industrial policies, such as state aid, we conclude that the fiscal authorities should be very careful when planning policies to support investment, especially in areas where the underground economy is sizeable, because of their perverse effects. **Figure 5** shows that capital subsidies have a deep impact on capital accumulation, but they also produce a reduction in the honest use of capital. In designing policy subsidies to the stock of capital, what should be taken into account is the "nature" of the firm, and, in particular, whether in the sector, and also in the area where the firm operates, a large part of output is unreported.

As incentives to investment in the presence of moonlighters always produce incentives to go underground, it may be argued there is the risk that this policy proves, via underground activities, to be what is known as a time-inconsistent policy. If government policies support moonlighting firms, the latter will not find it worth increasing their reported capital, given that they already enjoy fiscal incentives. Moreover, as we showed in section 3, technology (i.e. the TFP effect) matters for determining the extent of the declared production. Granting subsidies might have the side-effect of lowering the profitability to invest in TFP enhancing activities, which are a powerful mechanism to attain more official production.[28] Therefore, the government would be forced to subsidize capital accumulation for longer than expected, without having any considerable impact on tax evasion.

5 Conclusions

In this paper we have investigated the effects of selected fiscal policies, some of them broadly included among State aid policies, in a context in which underground production is feasible. The innovation of the paper is twofold: first, we represent a specific technological advantage (aggregate capital externality) of moonlighting firms over ghost firms that we believe is intuitively plausible and squares with common knowledge. Second, we consider the implication of this framework for two commonly used modes of fiscal policy designed to increase investment and the capital density of production (and, hence, ultimately labor productivity and standards of living).

In this regard, we set out an optimal investment model in which a representative firm maximizes its expected cash flow, by choosing an optimal combination of capital stock and its allocation between official and unofficial production. The model provides several striking policy implications.

First consider the troubling aspects of capital subsidies. In the context of moonlighting firms, which display a kind of technology that is certainly not purely theoretical, but is strongly supported by empirical evidence, government fiscal support of private investment spending yields an incentive not only to increased capital accumulation but also to its underground use. This policy is clearly counterproductive if a government also aims to reduce tax evasion.

The second result concerns the trade-off related to the tax-enforcement system. Though tax reduction and strict enforcement both reduce firms' incentives to operate underground, their effects on total production are basically different: while a tax reduction is also output improving, the enforcement policy yelds an output reduction. Thus, a trade-off arises between officially deployed capital share and firm size when unofficial production is combatted through rising expected penalties.

Third, under plausible calibration, there is evidence of a persistence in tax evasion which cannot be easily reduced under a threshold of about 15% without dramatic policies, such as very big tax cuts.

This last implication, largely consistent with both micro- and macro-founded evidence for Italy, suggests that a strategy against tax evasion should be multi-faceted, and strictly connected with industrial policies. The calibrated model suggests that in order to empower the official economy it is necessary to pursue a mixed strategy, aiming to reduce the fiscal burden, but also to raise firm efficiency. In fact, in our simulation it is crucial to improve the TFP, since a process of technological growth is very successful to promote official production.

Notes

[1]Estimates of the underground economy are particularly difficult as the phenomenon is, by definition, not directly measurable. Several methods have been used for this purpose, some based on theoretical models, some based on econometrics and others on micro analysis of agent responses in particular surveys. See, among others, The Economic Journal (1999) symposium on the Hidden Economy, and Busato, Chiarini and Di Maro (2006).

[2]We stress that in this paper tax evasion is related to underground activities, which are otherwise legal but go unreported or unrecorded.

[3]In Italy, the National Institute of Statistics (ISTAT) produces several time series estimates of the underground economy and employment, disaggregated at regional level since 1995. According to ISTAT, underground economy accounted, in 2006, for 17% of total Value Added. Data on tax evasion in Italy are currently provided by the Revenue Agency of the Ministry of Finance, which has recently estimated a yearly time series of the non-reported Value Added Tax base. The size of VAT evasion in percentage terms was about 30% in 2004.

[4]It is worth stressing that this evidence is affected by the sample selection, which is biased toward large-sized firms where fiscal controls guarantee more cash revenues.

[5]The survey carried out by Censis (Centre for Social Studies and Policies) is based on qualitative methodology, namely interviewing selected witnesses (managers, union representatives, public officers and so on).

Hibbs and Piculescu (2008), using data from the World Bank, point out that managers of more than 60% of 3,818 interviewed enterprizes, distributed over 54 countries, are used to operating both in the official and unofficial sector.

[6]European Union (EU) State aid policy is strictly regulated since it may harm competition. To this end, the European Commission Treaty (Art. 87) obliges EU governments to negotiate their allowances with the European Commission. There are several instruments of State aid (see Nitsche and Heidhues, 2006 and European Commission, 2006). *Grants* and *Tax Exemptions* and *Equity Participation* comprise aid that is transferred in full to the recipient and accounts for the vast majority of aid in all Member States. *Soft Loans* and *Tax Deferrals* cover transfers in which the aid element is the interest saved by the recipient during the period in which the capital transferred is at his/her disposal. *Guarantees*, expressed in nominal amounts guaranteed, incorporate aid elements corresponding to the benefit which the recipient receives free of charge or at lower than the market rate.

[7]See Cowell (1990).

[8]Solving Eq. 1 to show the separate contribution of the two productions we get:
$(1-\tau)\,Y^R + (1-\rho\tau s)\,Y^U$.

[9]In this paper we use a simple tax evasion model. There are many issues, concerning the penalty rate, the possibility of detection and audit, that we cannot discuss here. See, among others, the survey of Andreoni, Erald and Feinstein (1998), Slemrod and Yitzhaki (2002), Bayer (2006) and Sandmo (2006). Both the empirical and theoretical literature usually consider taxation and regulation as the main causes of the existence of the underground sector (see Thomas, 1992; Tanzi, 1980; Dallago,1990). Analysis of tax evasion, starting from Allingham and Sandmo (1972) and Yitzhaki (1974), focuses on the structure of marginal taxation, and/or the consequences for private/social welfare, without investigating the link between tax evasion and technology (see Cowell, 1990; Trandel and Snow, 1999, for surveys on tax evasion, and Alm, 1985 for the welfare effects of evasion). On the other hand, when focusing on the technology of underground activities, the literature very often concentrates on labour inputs, neglecting capital utilization (see Portes, Castells and Benton, 1989; Boeri and Garibaldi, 2001; Busato and Chiarini, 2004; Busato, Chiarini and Rey, 2005).

[10]Typical explanations include lower entrepreneurial ability, difficulty in getting financial support and high transaction costs due to the necessity to locate "trustworthy" trading partners. See Anderberg et al. (2003); Loayza (1994).

[11]In a different framework, Hibbs and Piculescu (2008) allow "B" to denote the productive value of institutional services available only to official activity. Here we claim it is more plausible that some of those services can be used surreptitiously to enhance unofficial production (through the moonlighting effect) without the enforcement authorities catching on, whereas totally unofficial ("ghost") firms do not have access to this profit enhancing externality.

[12]For greater details for Italy see Lucifora (2003) and Roma (2001).

[13] In the appendix it is shown that a sufficient condition to allow saddle path stability is: $\sigma < (a-1)^2 /a < (1-a)/a$.

[14] In order to ensure the advantageousness of the moonlighting technology, a sufficient condition is that Y(ghost)<Y(moonl) for a given capital stock, which reads as follows:

$K^{a\sigma}/B\,(1-\mu^a) > 1-\mu^a$.

This condition holds in all the parametrizations we use to define the steady state.

[15] We do not consider the effect of corruption and bribing activities, which are additional costs for firms operating in the underground economy. This issue is well addressed in Hibbs and Piculescu (2008).

[16] Output elasticity is the same in the two productions when considering the share of capital allocated there, μK and $(1-\mu)K$, whereas output elasticity of total capital K is larger in unofficial production due to the moonlighting effect.

[17] This assumption along with Condition 2 in the main text is a strong incentive toward underground production. A different situation would occur if the fiscal authorities were more effective in allowing incentives to capital than in detecting tax evasion. In this case the rational agent would choose to produce irregularly, $Y_U > 0$, but seek incentives only on the regular share of its investment, $\alpha\mu I$. This hypothesis complicates the analysis considerably, generating unstable and oscillating equilibria.

[18] The optimization problem is well defined, i.e. the objective function is concave:

$\partial^2\Pi/\partial^2\mu, \partial^2\Pi/\partial^2K, \partial^2\Pi/\partial^2I < 0$.

[19] The negative relation expressed by the $\mu(K)$ equation in System 13 is a consequence of the endogenous TFP in underground technology.

[20] Some insights into the size of the underground economy in EU countries are given in the Flynn Report (European Commission, 1998), which estimated that unofficial production of goods and services in European countries ranged between 7 and 16% of total GDP, with considerable differences among States. The largest shares of informal activities were recorded in southern countries, in particular Italy and Greece, with a size of about 20% of GDP, followed by Belgium and Spain, with slightly lower figures. More recent figures seem to confirm this ranking of countries (Schneider and Enste, 2002).

[21] For the size of the underground economy, see, Baldassarini and Pascarella (2003), Schneider and Enste (2002), and Busato, Chiarini and Di Maro (2006). An outline of the State aid to firms in Italy may be found in Bosco (2002) and Ministero delle Attività Produttive (2005).

[22] Calibration of the fiscal parameters τ and α was chosen starting from the analysis of the Italian firm fiscal regimes addressed in Bontempi et al. (2001). In particular, incentives to investment identified as *Credito di Imposta* ranges from an average level of 0.14 for the Center-North regions, to 0.65 for the least developed region (Calabria). As for corporate taxation, Chiarini et al. (2008) investigate the size and dynamics of two tax rates: the average "apparent" tax rate, where tax revenues are compared with the total GDP (reported and concealed), and the "effective" tax rate, where tax receipts are compared with the GDP net of a proxy of the concealed output. Finally, the calibration for the penalty, s, is dicussed in Busato Chiarini (2004).

[23] We simulate the outcomes for different values of B in Section 3.4.4.

[24] As stated above, we have a non-linear system described in a three-dimensional space. In order to find the steady state characteristics we must calibrate and simulate the system.

[25] Of course, every path other than the saddle path takes the firm far from the long-run equilibrium to areas in which the transversality condition (Eq. 7) no longer applies.

[26] Given Condition 1 in the main text, a fall in the tax rate necessarily causes, in the steady state solution (13), a fall in the taxation ratio $(1-\rho\tau s)/(1-\tau)$.

[27] Graphically, the locus $\dot{K} = 0$ would have the usual increasing shape, but we would also observe an increasing locus $\dot{\phi}_0 = 0$ situated above the $\dot{K} = 0$ such that no equilibrium could be found.

[28] See, for instance, Tornell's (1991) analysis for trade policies.

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6 Appendix

Proposition 2 *In the long run, the dynamic system pertaining to steady state System 10 admits a unique steady state.*

Proof. *System 10 can be written as follows:*

$$\begin{cases} \phi_0 = \frac{1}{(r+\delta)}\left[(1-\tau)aB\left[\mu^*(K)\right]^a K^{a-1} - r(1-\tau) + (1-\rho\tau s)\left[1-\mu^*(K)\right]^a a(1+\sigma)K^{a(1+\sigma)-1}\right] \\ \phi_0 = 1-\alpha+b\left(\delta K\right)^{b-1} \\ \mu^*(K) = \frac{CK^d}{1+CK^d} \end{cases}$$

First Step:

To show that the first equation expresses ϕ_0 as a monotone and decreasing function of the stock of capital K .

$$\frac{d\phi_0(K)}{dK} = \varphi\left[(a-1)\left[\mu^*(K)\right]^a K^{a-2} + a\mu^*(K)'\left[\mu^*(K)\right]^{a-1}K^{a-1}\right] +$$

$$+\chi\left[(a(1+\sigma)-1)(1-\mu^*(K))^a K^{a(1+\sigma)-2} - a\left[1-\mu^*(K)\right]^{a-1}K^{a(1+\sigma)-1}\mu^*(K)'\right]$$

$$\varphi = \frac{(1-\tau)aB}{(r+\delta)} > 0; \chi = \frac{(1-\rho\tau s)a(1+\sigma)}{r+\delta} > 0.$$

The expression derived from equation 7 in the main text:

$$\mu^*(K) = \frac{CK^d}{1+CK^d}; d = \frac{a\sigma}{a-1} < 0; C = \left(\frac{(1-\rho\tau s)a}{(1-\tau)aB}\right)^{\frac{1}{a-1}} > 0$$

is a strictly decreasing and monotone function of K:

$$\frac{d\mu^*(K)}{dK} = \frac{CdK^{d-1}\left[1+CK^d\right]-CdK^{d-1}\left(CK^d\right)}{\left[1+CK^d\right]^2} = \frac{CdK^{d-1}}{\left[1+CK^d\right]^2} < 0 \quad \forall a < 1$$

It implies that the first term of $\frac{d\phi_0(K)}{dK}$ is always negative, such that we need to demonstrate that the second one is negative too:

$$\left[a(1+\sigma)-1\right]\left[1-\mu^*(K)\right]^a K^{a(1+\sigma)-2} - a\left[1-\mu^*(K)\right]^{a-1}K^{a(1+\sigma)-1}\mu^*(K)' < 0$$

Using again the definition of $\mu^*(K)$ as well as $\frac{d\mu^*(K)}{dK}$ we get:

$$\left[a(1+\sigma)-1\right]\left[\frac{1}{1+CK^d}\right]^a K^{a(1+\sigma)-2} - a\left[\frac{1}{1+CK^d}\right]^{a-1} K^{a(1+\sigma)-1}\frac{CdK^{d-1}}{\left[1+CK^d\right]^2} < 0$$

$$\left[a(1+\sigma)-1\right]\left[\frac{1}{1+CK^d}\right]^a K^{a(1+\sigma)-2} - aK^{a(1+\sigma)-2}\frac{CdK^d}{\left[1+CK^d\right]^{a+1}} < 0$$

$$\left[a(1+\sigma)-1\right]\left[\frac{1}{1+CK^d}\right]^a - a\frac{CdK^d}{\left[1+CK^d\right]^{a+1}} < 0$$

$$\left[a(1+\sigma)-1\right] - a\frac{CdK^d}{\left[1+CK^d\right]} < 0$$

$$\left[a(1+\sigma)-1\right] - ad\mu < 0.$$

As μ is a majorant of this last expression, we consider the case $\mu = 1$ to get:

$$\sigma < (a-1)^2/a < (1-a)/a$$

This condition can be considered a sufficient condition to obtain the required monotony of the relation $\dot{\phi}_0 = 0$.

Second Step

The second equation expresses ϕ_0 a monotone and increasing function of the stock of capital K. Indeed, $\frac{d\phi_0}{dK} = b\,(b-1)\,(\delta K)^{b-2} > 0$ for each $b > 1$.

Given that the codomain of the first equation is $(0; +\infty)$ while the second equation has codomain $(1-\alpha; +\infty)$, it follows that there exists a single value of K such that the two equations simultaneously apply. ■

Proposition 3 *The steady state of the dynamic system pertaining to steady state System 10 is always a saddle path.*

Proof. The dynamic system related to the steady state System 10 can be written as follows:

$$\begin{cases} \dot{\phi}_0 = (r+\delta)\,\phi_0 - (1-\tau)aB\,(\mu^*)^a\,K^{a-1} + r(1-\tau) - (1-\rho\tau s)\,[1-\mu^*]^a\,a(1+\sigma)K^{a(1+\sigma)-1} \\ \dot{K} = I\,(\phi_0) - \delta K \end{cases}$$

The Jacobian of this System of Equations evaluated at the steady state is:

$$\begin{bmatrix} r+\delta & -\partial^2\Pi/\partial^2 K \\ \partial I/\partial\phi_0 & -\delta \end{bmatrix}$$

and it has a trace and a determinant given by:

$TR = r; DET = -\delta\,(r+\delta) + \partial I/\partial\phi_0\,\left(\partial^2\Pi/\partial^2 K\right);$

where

$\partial^2\Pi/\partial^2 K = d\left[(1-\tau)aB\,(\mu^*)^a\,K^{a-1} + (1-\rho\tau s)(1-\mu^*)^a a(1+\sigma)K^{a(1+\sigma)-1}\right]/dK$

Local stability, and in particular saddle path stability, requires that the trace should be positive, while the determinant should be negative, when evaluated at the steady state. Under our parametrization it implies that the condition $\partial^2\Pi/\partial^2 K < 0$, which is the necessary condition to get a concave objective function, is also a sufficient condition to get saddle path stability. Given the demonstration of the first step of Proposition 2, it follows that

$\partial^2\Pi/\partial^2 K < 0$.

This result implies that the Determinant of Jacobian matrix of linearized System 10 is negative, and it underlines that the equilibrium is a saddle path. ■