An evolutionary model of firms location with technological externalities

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Abstract

We introduce an evolutionary model of firms location, and use it to characterize the long run geographical distribution of firms. We consider an entry-exit process of firms, whose locational choice is driven by a common and an idiosyncratic component. A characteristic of our model is that the common, location specific, component is shaped both by negative pecuniary externalities and by positive technological externalities. We show that, dependently on the value of transportation costs, either one or the other effect prevails. That is, the overall firms dynamics leads to equidistribution when transportation costs are high and to agglomeration, when transportation costs are low. Notice however that when agglomeration occurs, it is only a metastable state due to the stochastic nature of the dynamics.

1 Introduction

It is doubtless that the dynamics of all social systems, also those based on economic relations and interactions, is characterized by a continuous and restless production of novelty. While this fact has since long received ample recognition, starting with the classic Adam Simth and Karl Marx to Joseph Schumpeter, the economic discipline is still struggling in search of a proper way to measure it and to capture its essence in theoretical models. The difficulties of assessing, inside a broadly defined economic system, the effects and sources of "change" mainly reside in the pervasive nature of the latter. The way in which goods are produced, received and selected by consumers and, not to a lesser extent, the way in which these goods are actually traded are constantly changing. An ubiquitous and pervasive process does indeed shape the organization of market institutions, modify the terms and conditions of the networks in which production is carried over and the way in which producers try to match consumers' tastes and preferences.

How to correctly describe these ever changing environment and how to properly introduce the dynamics of innovation in economic theorizing has been one of the key issues, if not "the" key issue, of the evolutionary economics tradition (Nelson and Winter, 1982; Freeman, 1986; Dosi, 1988). One could safely say that the main idea of this approach rests in the assumption that the dynamics of change and the creation of novelty can be described as a process of "evolution", broadly in line with the equivalent notion emerged two centuries ago in the life sciences (Gould, 2002).

In principle, the validity of the assumption of evolutionary economics is all but obvious and the question whether economic change can be effectively thought as an evolutionary process still open. With the clear risk of oversimplifying the matter, we could say that the notion of evolution immediately entails three consequences for the economic dynamics. First, it should move from simpler to more complex structures. Second, it should progressively eliminate less efficient structures and promote the development of more efficient ones, irrespectively of the fact that this process of elimination and promotion might take place through a mechanism of adaptation by part of the economic actors or through an"adoptation" by part of markets and institutions (Alchian, 1950). Third, the progressive change or renewal of the different actors and rules should proceed in a jointly integrated way.

Obviously, the central question is not whether the characteristics described above can be considered pervasive components of economic systems, because they quite probably are. The question is whether the evolutionary accounting of their effects and causes allows for a deeper understanding and a more reliable modeling of economic interactions. In the end, one is interested to know if this accounting could help in the development of more effective policies. Practically, however, in order to apply the ideas of evolutionary economic thinking to the investigation of the different domains of economics, one has not to provide a certain and undisputable answer to the previous questions. Indeed, partly following, even if not subscribing to, the Friedmanian's idea that the effectiveness of a theoretical framework should be solely judged on its ability to reproduce and explain observed phenomena, one could simply start from the "evolutionary" metaphor and see what consequences it brings to the design of economic models.

In the spirit of the foregoing "minimalistic" research agenda, the present contribution intends to pursue the analysis of the effects of the evolutionary metaphor when applied inside the domain of economic geography. Indeed, as argued in Frenken and Boschma (2007), the development of an evolutionary approach to economic geography could suggest new ways of explaining the observed patterns which characterize the uneven spatial distribution of economic activities. Fundamentally, our aim is to complement the bottom-up theorizing suggested there with a deeper understanding of the differences that an evolutionary inspired modeling is likely to produce with respect to more traditional approaches.

To be brought to its completion our exercise requires a twofold specification. First, we need to identify a simple formal model, based on clear assumptions, which can serve as a generic analytical framework. Second, we have to consider which hypotheses are to be put forward in order to imbue this model with the spirit of the Evolutionary Economic Geography. We address the first requirement choosing, as a starting point, the simple two-location and multi-firm model described in Krugman (1991). This model already encompasses the idea of increasing returns and of the relevance of feed-back mechanisms in shaping the aggregate economic pattern. It is well rooted in the tradition of New Economic Geography and, as such, constitutes a perfect benchmark for our comparative exercise. Concerning the second requirement, and in line with the discussion in Boschma and Frenken (2006) and Boschma and Martin (2007), we assume the following three aspects as baseline characters of our evolutionary modeling. First, the interaction between economic agents should take place not only through market mechanisms, but also through localized, idiosyncratic interactions. Second, the flow of time should be present in the model and the decision of economic agents, together with their consequences, should be put in an explicit time dimension. Third, the heterogeneity of firms behavior should not be captured by a simple "noise" term acting as a perturbation around a deterministic equilibrium. Rather, it should enter as an essential ingredient in the description of the model and in the determination of the final aggregate outcome (Schelling, 1978; Granovetter, 1978).

More precisely, we take as a starting point the model introduced in Bottazzi and Dindo (2008) where production and consumption take place in different geographical regions, or locations. This model extends Krugman (1991) by introducing a positive technological externality, assumed not tradable across locations, and by considering workers who are not mobile. The latter extension is equivalent to assume that the firms locational decision and the reallocation of capital goods take place on a much shorter time scale than the one characterizing workforce flows. Inside this simple economy, we consider an heterogeneous population of profit maximizing firms which independently choose where to locate their production. The model is characterized by a simple entry-exit process, and we consider a truly dynamic setting in which the locational decision of each firm is affected by the previous decision of others. In fact, firms keep revising their decision as new realizations of location choices affect their profit.

The idea that localized externalities might explain agglomeration even in absence of workers mobility, has been explored by several contributions inside the New Economic Geography literature. For instance Krugman and Venables (1996) assume a vertically structured economy with localized input-output linkages while Martin and Ottaviano (1999) consider location-specific R&D sectors that introduce different products in different locations. A drawback of these works is that, in general, they derive equilibria conditions without the complete and explicit characterization of the firms profit function. This specification is however necessary in order to design firms choice procedure in a dynamic environment. In order to obtain explicit expression for the profit function, we take a simpler approach: we introduce a technological externality in the forms of a baseline "cost sharing" assumption, according to which fixed production costs are shared across all firms within a given location.

The assumption above makes the model in Bottazzi and Dindo (2008) particularly suitable for the present exercise because, while remaining simple and analytically tractable, it allows for a twofold dependence of firm profits on the activity of the other firms. Using the terminology of Scitovsky (1954), this dependence takes the form of both a pecuniary externality and a technological externality. In this way, firms profits will be dependent on the interplay of an indirect interaction mediated by the market, which corresponds to a pecuniary externality, and a localized direct interaction, which corresponds to a technological externality. As it turns out, in a partial equilibrium setting with constant wages, the former acts against the creation of production clusters while, by assumption, the latter promotes them.

Inside this framework, we analyze the evolutionary firms locational decision process. Our first aim is to characterize the long run geographical distribution and relate it to the interplay of the two forms of externality. Since we explicitly introduced the time dimension in our analysis, we are also able to address history dependent phenomena. In particular, we are able to investigate how the initial state of the economy affects firms decision and show that, due to heterogeneity of firms, when agglomeration occurs it is characterized by a transient nature.

This chapter is organized as follows. In Section 2 we briefly describe the model and its assumption. In Section 3 we study the static settings, and derive the geographical distribution when the model is solved by assuming instantaneous equilibrium between firms choices.

Starting from the previously identified equilibria, Section 4 introduces both heterogeneity in agents decisions and an explicit dynamics across time, discussing what kind of differences are observed with respect to the static case. Finally Section 5 summarizes our main findings and suggests some possible further developments.

2 The model

Consider a simple two locations economy. Location $l = 1, 2$ has n_l firms and I agents so that the total number of firms is $n_1 + n_2 = N$ and the total number of agents $2I$.[1] Each agent is a "local" worker, that is he supplies labour to firms located where he resides, and a "global" consumer, that is he can buy goods produced in both locations and traded in a global market. Each firm produces a different product, so that there are N products traded in the global market and available for consumption. In order to consume goods produced in the location where they do not reside, consumers have to pay a transportation cost $\tau \in (0, 1]$ which takes the form of an iceberg cost: for each unit of good shipped, only a fraction τ arrives at destination. This is equivalent to say that consumers pay a price p_i/τ for each unit of good $i = 1, \ldots, N$ which they have to import. The higher the value of τ, the lower the cost of transporting goods. Agents consumption behavior is specified by the following

Assumption 1. Each agent chooses among the N different products as to maximize a Constant Elasticity of Substitution utility function (CES) of the type

$$\mathrm{U}(c_1, \ldots, c_N) = \left(\sum_{i=1,N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{(\sigma-1)}} \qquad \sigma > 1 \,, \tag{2.1}$$

where c_i, $i = 1, \ldots, N$, is produced by firm i and $\sigma \in (1, +\infty)$.

Assumption 1 implies that the N products are substitutes and that σ is the mutual elasticity of substitution (cfr. Dixit and Stiglitz, 1977). The higher σ, the more the products are substitutes and the more price differences between products impact consumers demand.

Firms are profit maximizers and produce using only labor as input. Their technologies are characterized by a common, industry specific, marginal cost and a location specific fixed cost. Formally one has

Assumption 2. The labour v_i that each firm $i = 1, \ldots, N$ needs to produce an amount y_i of output is given by

$$v_i = \beta y_i + \alpha_{l_i} \,, \tag{2.2}$$

where the production costs β are constant across firms and across locations and the fixed costs α_{l_i} depend on the location l_i of firm i.

Given the profit maximization behavior of firms and the structure of preferences in Assumption 1 the market structure is that of monopolistic competition: given agents inverse demand, each firm sets the output so that marginal revenues are equal to marginal costs.

[1] The generalization to different number of agents in each location will be considered in further work. Preliminary analysis shows that this generalization doesn't significantly modify the results. As far as the total number of firms is concerned, without loss of generality, we assume that N is even.

Notice that Assumption 2 implies that we are in presence of economies of scale, that is, an increase in output causes a decrease in each firm average costs. Firm i profit is given by

$$\pi_i = p_i y_i - w_i v_i \,, \quad i = 1, \ldots, N \tag{2.3}$$

where w_i is firm i cost of labour.

In order to find equilibrium prices, quantities, profits and the resulting geographical distribution of firms, one should in principle analyze each of the N product markets. Nevertheless the problem can be simplified by considering only a representative market for each location. In fact, location by location, firms produce using the same technology, face the same demand (due to Assumption 1 all goods are substitutes), and the same labour supply. This implies that equilibrium prices, quantities and wages are the same for all the firms in a given location. We can thus consider only two representative product markets, one for each location l, rather than the N distinct products. Concerning the costs of input we have the following

Assumption 3. Nominal wages are equal between the two locations, that is $w_1 = w_2 = w$. Furthermore the economy is in a state of full employment and agents are not mobile, that is, each location has a fixed number of agents I.

Assumption 3 deserves some comments. Equal nominal wages can be justified if, for example, there are unions operating across the locations and asking for equal wages on paper. However, even if nominal wages are the same in both locations, agents might still want to move in order to maximize the real, rather than the nominal, income. Real income would be determined by a price index which takes into account the fact that imported goods are more expensive than locally produced ones. By assuming that the number of agents in each location is constant and equal to I, so that they cannot move and change location, we rule out the possibility that they react to price differentials. A possible justification would be that, although real wages differ, agents do not change location due to moving costs and a very high discount factor so that higher future real wages are not enough to offset the effort to move. More generally, even if one considers agents who could relocate, as long as they do it at a time scale longer than firms do, our assumption of no agents mobility is still a good approximation of the economy's short term dynamics. By further assuming full employment at this wage we assume that agents are working either in one of the firms located in their region or in a different sector, e.g. agriculture, which we do not model, that absorbs excess supply of labor. Basically, we are in a partial equilibrium settings in which the reallocation of capital goods and human resources takes place on different time scales.[2]

Using the assumptions above we are ready to explicitly compute market equilibrium prices, quantities and profits for each fixed distribution of firms, that is, fixed n_1 and n_2. First, exploiting the CES preference structure (2.1), which gives us an a priori known demand elasticity, we derive firms pricing behavior. Then we compute agents total expenditure for the goods produced in each location, taking into account that all goods are substitutes and transportation costs impact the prices of foreign goods. Setting revenue equal to expenditure, which is equivalent to setting supply equal to demand, and using agents budget constraints, we are able to determine equilibrium quantities and firms profits in each location as a function of n_1 and n_2. These expressions will be used, in the next section, to asses firms geographical distribution.

[2]The generalization of the present framework to encompass also a second sector can be found in Bottazzi and Dindo (2008), where a general equilibrium version of the present model is analyzed. Since results from both settings are similar, but the partial equilibrium approach is easier to analyze, in this chapter we opt for the latter.

Let us start from firms pricing behavior. As implied by our assumptions the market structure is that of monopolistic competition, where each firm sets its marginal revenue equal to its marginal costs. In location l this gives

$$p_l \left(1 + \frac{1}{\varepsilon}\right) = \beta w \,, \tag{2.4}$$

where $\varepsilon = \partial \log c / \partial \log p$ is the demand elasticity. Given Assumption 1, as long as the number of commodities N is large (see Dixit and Stiglitz, 1977, for the details), it holds that

$$\varepsilon = -\sigma,$$

which together with (2.4) implies

$$\frac{p_l}{w} = \beta \frac{\sigma}{\sigma - 1} \,. \tag{2.5}$$

As a result the revenue of a firm in l is given by

$$\frac{p_l y_l}{w} = \beta \frac{\sigma}{\sigma - 1} y_l \,, \tag{2.6}$$

and, using (2.3), its profit is

$$\frac{\pi_l}{w} = \frac{\beta}{\sigma - 1} y_l - \alpha_l \,, \tag{2.7}$$

where we have used the wage w as a normalization factor.

Denote now the quantity consumed by an agent who reside in l of a product produced in m as d_{lm}. Equating, location by location, firms revenue and consumers expenditure for the goods produced in the corresponding location, we get

$$\begin{cases} n_1 p_1 y_1 &= I n_1 d_{11} p_1 + I n_1 d_{21} \dfrac{p_1}{\tau} \,, \\[2ex] n_2 p_2 y_2 &= I n_2 d_{12} \dfrac{p_2}{\tau} + I n_2 d_{22} p_2 \,. \end{cases} \tag{2.8}$$

Notice that prices of imported goods take into account the transportation cost τ. In order to solve (2.8) for y_1 and y_2 we need to know agents demand. To this purpose we use the fact that relative demand under a CES utility satisfies

$$\frac{d_{11}}{d_{12}} = \left(\frac{p_2}{p_1 \tau}\right)^{\sigma} \,, \quad \frac{d_{22}}{d_{21}} = \left(\frac{p_1}{p_2 \tau}\right)^{\sigma} \,, \tag{2.9}$$

while agents budget constraint gives

$$\begin{cases} w = n_1 d_{11} p_1 + n_2 d_{12} \dfrac{p_2}{\tau} \,, \\ w = n_1 d_{21} \dfrac{p_1}{\tau} + n_2 d_{22} p_2 \,. \end{cases} \tag{2.10}$$

Dividing both l.h.s and r.h.s. of (2.8) by the level of wages w, using (2.10), (2.9), (2.6), and noticing that (2.5) implies $p_1 = p_2$, one can solve for the market equilibrium quantities and gets

$$\begin{cases} y_1 &= \dfrac{\sigma - 1}{\beta \sigma} \left(\dfrac{I}{n_1 + n_2 \tau^{\sigma - 1}} + \dfrac{I}{n_1 + \frac{n_2}{\tau^{\sigma - 1}}} \right) \,, \\[3ex] y_2 &= \dfrac{\sigma - 1}{\beta \sigma} \left(\dfrac{I}{n_2 + \frac{n_1}{\tau^{\sigma - 1}}} + \dfrac{I}{n_2 + n_1 \tau^{\sigma - 1}} \right) \,. \end{cases} \tag{2.11}$$

Profits in each location can now be found using (2.7) and (2.11). Introducing the fraction of firms in location 1, $x = n_1/N$ (so that $n_2 = (1-x)N$), and normalizing the level of wages to one, profits can be finally written as a function of x

$$\begin{cases} \pi_1(x) &= \dfrac{I}{N\sigma}\left(\dfrac{1}{x+(1-x)\tau^{\sigma-1}} + \dfrac{\tau^{\sigma-1}}{x\tau^{\sigma-1}+(1-x)}\right) - \alpha_1\,, \\ \pi_2(x) &= \dfrac{I}{N\sigma}\left(\dfrac{1}{x\tau^{\sigma-1}+(1-x)} + \dfrac{\tau^{\sigma-1}}{x+(1-x)\tau^{\sigma-1}}\right) - \alpha_2\,. \end{cases} \tag{2.12}$$

Each location specific profit function in (2.12) has a positive term proportional to the global demand for good produced in that location and a negative term equal to the location specific fixed costs. In turn, the total demand in (2.11) has a domestic component, first term in parenthesis, and an export component, second term in parenthesis. Both components depend on firms geographical distribution and transportation costs. When transportation cost is zero, $\tau = 1$, they are equal irrespectively of firms distribution. When transportation cost increases, the domestic component increases, as local consumers substitute foreign goods with local one. For the same reason, the export component decreases. For any given positive transportation cost, when local firm concentration increases, the local component decreases as agents have more local goods to consume, all at the same price. In the same situation the export component increases, because foreign consumers have less local goods to consume and find convenient to import more. The net effects of the transportation cost on the relative profits of the two locations are appraised in the next section. However, even without knowing this effect, it is immediate to see that market forces act in such a way to make the average profits independent on transportation costs. Indeed one has the following

Proposition 2.1. *Consider an economy with two locations, $l = 1, 2$, and N firms, where Assumptions 1-3 are valid. Then the average firms profit $\bar{\pi}$ does not depend on transportation costs and it is given by*

$$\bar{\pi} = \frac{2I}{N\sigma} - x\,\alpha_1 - (1-x)\,\alpha_2\,. \tag{2.13}$$

Proof. See Appendix. □

If one assume that $\alpha_1 = \alpha_2$, since location specific indexes have disappeared from any variable, only market forces are at work and our model becomes close to the one of Krugman (1991).[3] We shall follow a different specification.

2.1 Technological externalities

We introduce a localized positive externality in the form of a technological externality (Scitovsky, 1954). To this purpose we can retain a dependency of the fixed cost on the location index. In this way we have a term of direct firms interaction, that is, not mediated by market forces.

[3]Notice, however, that Krugman (1991) assumes that workers can move between the two regions in the search for higher real wages.

Assumption 4. "Cost sharing" hypothesis. Firms fixed costs α_l decreases with the number of firms located in each region according to

$$\alpha_l = \frac{\alpha}{2x_l}\,. \tag{2.14}$$

Assumption 4 represents a positive technological externality in the form of a baseline "cost sharing", so that the larger the number of firms in one locations, the lower the fixed costs these firms bear in the production activity. More specifically, the fixed cost payed by firms in a given location decreases proportionally with the number of firms populating that location, so that the total fixed cost payed remains, location by location, constant. This effect can be thought as an up front cost payed to improve access to skilled labour, the more firms in one location the smaller each firm investment in training, or as a cost for services or infrastructure use, which are evenly shared among all the active firms in one location.

An important feature of the specific form of "cost sharing" introduced in Assumption 4 is that it doesn't modify the total fixed costs payed by the industry. This has a consequence on the computation of firms average profit.

Corollary 2.1. *Consider an economy with two locations, $l = 1, 2$, and N firms, where Assumptions 1-4 are valid. Total costs in each location are equal to $\alpha N/2$. The average firms profit $\bar{\pi}$ does not depend neither on firms distribution x nor on transportation costs and is given by*

$$\bar{\pi} = \frac{2I}{N\sigma} - \alpha\,. \tag{2.15}$$

Proof. See Appendix. □

Before we start to look for geographical equilibria, that is, those spatial distributions of firms where there are not incentives to change location, notice that, without restriction on the parameters values, there could exist economies characterized, even at equilibrium, by negative profits. In this case, we would expect firms to exit the economy. On the other hand, if equilibrium profits were positive we could expect firms to enter the economy. As we consider the number of firms N in the model as given, if there are not barriers to entry, it seems reasonable to set the number N to a level which implies zero equilibrium profits. By force of Corollary 2.1 this can be done also without knowing the equilibrium distribution. Indeed equilibrium profits are always equal to average profits and average profits (2.15), due to Corollary 2.1, are independent on the geographical distribution x. So it is enough to have the following

Assumption 5. The number of firms N is such that profits at a geographical equilibrium, π^*, are zero, that is,

$$N = \frac{2I}{\sigma\alpha}\,. \tag{2.16}$$

Even if, by construction, the previous assumption implies $\bar{\pi} = 0$, outside the equilibrium profits can be both positive or negative. When it is so profits differentials, which give firms the incentive to relocate, are still present. Before moving to the analysis of these incentives and to the characterization of the geographical equilibria, it is useful to rewrite profits (2.12) incorporating Assumptions 4-5

$$\begin{cases} \pi_1(x) &= \dfrac{\alpha}{2}\left(\dfrac{1}{x+(1-x)\tau^{\sigma-1}} + \dfrac{\tau^{\sigma-1}}{x\tau^{\sigma-1}+(1-x)}\right) - \dfrac{\alpha}{2x}\,, \\ \pi_2(x) &= \dfrac{\alpha}{2}\left(\dfrac{1}{x\tau^{\sigma-1}+(1-x)} + \dfrac{\tau^{\sigma-1}}{x+(1-x)\tau^{\sigma-1}}\right) - \dfrac{\alpha}{2(1-x)}\,. \end{cases} \tag{2.17}$$

Given the firms production costs α, the products elasticity of substitution σ and the transportation cost τ, the distribution of firms between the two locations, x, determines, through (2.17), the levels of profit. Notice that in (2.17), differently from (2.12), both the demand driven term and the fixed cost term are functions of the geographical distribution of firms. The first dependence is mediated by market forces (pecuniary externality) whereas the second dependence is brought in by the "cost sharing" hypothesis (technological externality).

3 Geographical equilibria

In this section we investigate the static geographical equilibria of the system, that is, those distributions of firms x such that, in the search for higher profits, each firm located in 1 has no incentive to move to 2 and vice versa. Geographical equilibria can be of two types: "border" equilibria and "interior" equilibria. A border equilibrium occurs when firms concentrate in one location, say 1, and profits in 1 are higher than profits in 2. As all the firms are in 1, no other firm can respond to this difference in profit opportunities. Candidates for border equilibria are $x = 1$, when all firms are in 1, and $x = 0$, when all firms are in 2. An interior equilibrium occurs when firms distribute among the two locations, that is $x \in (0, 1)$, but none of them has an incentive to change location because profit levels are equal in both regions. Using profits in (2.17) we will derive results for the existence and uniqueness of geographical equilibria, border and interior, for all the different specifications of the economy. This static[4] analysis, which owes considerably to Bottazzi and Dindo (2008) where more details can be found, is useful to understand the interplay of pecuniary and technological externalities and constitutes an useful step for the development of the evolutionary dynamic analysis of the next section.

The respective role of each externality in determining profit differential and thus the aggregate geographical equilibrium can be judged by looking at the shape of the profit functions. First consider the pecuniary externality terms, in parenthesis in (2.17). Notice that, due to transportation costs, local prices are lower than foreign price, and thus local demand impacts firms level of output more than foreign demand. For definiteness, consider profits in 1 (results for profits in 2 follows in the same way). For small x, that is few firms in location 1 and many firms in location 2, each firm in 1 faces a high local demand and a low foreign demand. Then the level of output of firmsin 1 is high and, due to (2.7), profits are high too. As x increases, the local demand for these firms decreases, so that profits decrease too. As the concentration of firms in 1 increases further, for sufficiently large value of x, the demand coming from the

[4]Technically a geographical equilibrium corresponds to a Nash Equilibria in Pure Strategies of the one shot game where each firm in a group of N has to chose whether to be located in 1 or in 2 and payoffs are given by profits.

consumers in 2, where very few firms are left, is more and more directed to 1 and the profits of firms located in 1 increase again. Profits are thus U-shaped, with $\pi_1(x)$ first decreasing and then increasing in x. Since firms make the most profits when they are alone in one location, we have $\pi_1(0) > \pi_1(1)$ so that the border distributions 0 and 1 are never an equilibrium. In fact, when all firm are located in one region is always more profitable to move to the other region. It the transportation cost is increased (decreased) the variation of profits as a function of x is more (less) pronounced but the general shape of the profit function is preserved. As a result, the overall agglomeration effect of the pecuniary externality is always "negative", in the sense that it works against concentration of production.

The above picture changes completely when one consider also the technological externalities terms introduced with the "cost sharing" assumption. The panels in Fig. 1 show graphs of $\pi_1(x)$ and $\pi_2(x)$ corresponding to the expression in (2.17). Profits are given by the superposition of monotonically increasing technological externalities to the U-shaped market driven pecuniary externality term. When transportation costs are low (high τ) the profit function is essentially determined by the "cost sharing" and is monotonically increasing with decreasing marginal profits (left-upper panels of Fig. 1). In fact, when firms concentration is low, firms do not benefit from the technological externality and their profits are low too, but as firms concentration increases, profits increase monotonically as firms exploit the "cost sharing" opportunity. Furthermore the more the firms in one location, the lower the positive contribution on an extra firm locating there, so that the marginal profit decreases. When transportation costs are high (low τ) the shape of the profit function is still monotonic (bottom-right panel of Fig. 1), but marginal profits are first increasing and then decreasing. With low firm concentration the technological externality dominates and profits are increasing with decreasing marginal profits. As the concentration increases, the positive effect of the cost sharing is almost offset by the negative market interaction, which act as a constraint on the local demand faced by firms. In this case, even if profits are still increasing the marginal profits is almost zero. As the concentration of firms increases further, profits increase more steadily because low local demand is now compensated by the foreign demand, so that the contribution of the pecuniary externality is positive too. Judging from Fig. 1, irrespectively of the transportation costs, the positive effect of technological externalities is dominating the negative effect of pecuniary externalities: firms make most profits by agglomerating on one side and border distributions are always equilibria. This is formalized in the

Proposition 3.1. *Consider an economy with two locations, $l = 1, 2$ where Assumptions 1-5 are valid. Call x the fraction of firm located in 1. There always exists two, and only two, geographical equilibria given by the border distribution $x_1^* = 1$ and $x_0^* = 0$. In particular, the unique distribution where profits are equal, $x^* = 0.5$, is never an equilibrium.*

Proof. See Appendix. □

According to the previous proposition, the distribution with half of the firms located in 1 and the other half located in 2, which is the unique point where $\pi_1 = \pi_2$, is never a geographical equilibrium: even if profits are equal, incentives are such that firms move away and agglomerate. Only when all firms are located either in 1 or 2 there are no incentives to change location.

Notice that, even if transportation costs do affect the shape of each location profit function, they have no impact in characterizing the geographical equilibria of the economy. Conversely, as we shall see in the following section, transportation costs do have a strong impact, even on the long run outcomes, in shaping the results the evolutionary model.

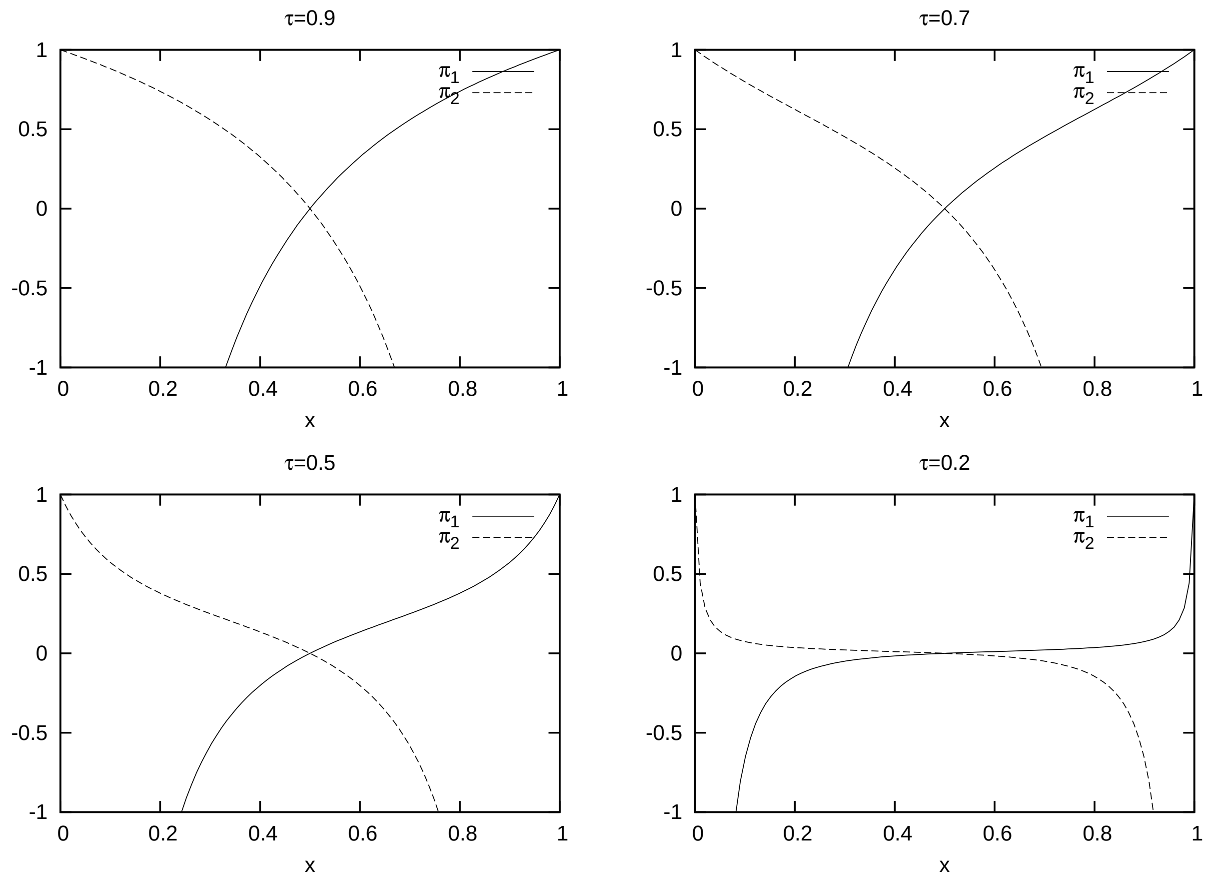

Figure 1: Location profits (2.17) as a function of firms geographical distribution for different values of the transportation cost τ. The other parameters are $\sigma = 4$, $\alpha = 1$, $I = 400$, $N = 100$.

4 Evolutionary firm dynamics

In the previous section we have shown that, when the technological externality term is introduced, firms agglomerate in one of the two locations, no matter the value of transportation costs or the other characteristics of the economy. The foregoing analysis was essentially static and thus silent on the results of firms interaction out of the equilibrium. As a consequence, it is not clear what happens when the initial concentration of firms is not at an equilibrium level, in particular, whether one should expect firms to agglomerate in location 1 or in location 2.

In this section we extend our anaysis by explicitly modeling firms decisions in time, that is, by allowing for a dynamic location selection mechanism. At the same time, we also introduce heterogeneity in preferences at the single firm level and characerize how it affects the overall dynamics of the economy, for example by checking whether firm dynamics is still converging to agglomeration. For these purposes, we complement the economic framework described in Section 2 with the discrete choice model outlined in Bottazzi and Secchi (2007); Bottazzi et al. (2007). At every time step a firm is randomly chosen to exit the economy. At the same time a new firm enters and chooses whether to be located in 1 or in 2 by comparing their relative utilities. These utilities are obtained summing the profit it can actually earn in each location, π_l, and a firm-specific component $e_{i,l}$ intended to capture its own particular preferences. As long as the distribution of $e_{i,l}$ across firms is well behaved (cfr. Bottazzi and Secchi, 2007, for details) the resulting probability of choosing l in place of m is given by

$$p_l = \frac{e^{\pi_l}}{e^{\pi_1} + e^{\pi_2}}\,, \quad l \in \{1, 2\}\,, \tag{4.1}$$

where profits π_l $l = 1, 2$ are as in (2.17). The fact that the locational choice is probabilistic derives from the assumption that the new entrant possesses preferences randomly extracted from a suitably defined set.[5]

When the probability of choosing location l is given by (4.1), Bottazzi and Secchi (2007) show that, if profits are linearly changing in the number of firms, it is possible to compute the long run stationary distribution of the entry-exit process. We shall use their result in order to investigate how the overall firms stochastic dynamics is influenced by the dominance of pecuniary externalities or technological externalities. For this purpose we need a linearized version of the profit functions that we obtain as deviation from the middle point $x^* = 0.5$, that is, the unique point where profits are equal.

Proposition 4.1. *Consider an economy with two locations, $l = 1, 2$, where Assumptions 1-5 are valid. Denote the linearization of location l profits around $x^* = 0.5$ as c_l, and the number of firms in location l as n_l. Linearized profits are given by*

$$c_l = a + b n_l\,, \quad l = 1, 2\,, \tag{4.2}$$

where

$$\begin{aligned} a &= 1 - \frac{4\alpha\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\,, \\ b &= \frac{4\alpha^2\sigma\tau^{\sigma-1}}{I(1+\tau^{\sigma-1})^2}\,. \end{aligned} \tag{4.3}$$

Proof. See Appendix. □

We shall call the term a in (4.3) the "intrinsic profit". This is the part of the common profit which is entirely dependent on exogenously given characteristics of the location. Conversely, the coefficient b in (4.3) captures the marginal contribution of a firm to the profit level of the location in which it resides. We shall call it the "marginal profit".[6] In our case, this coefficient captures the total effect of pecuniary and technological externality. Due to the leading effect of the latter it is always positive, but the presence of market mediated interactions makes it dependent on transportation costs. Specifically, the marginal profit is increasing with the value of τ, so that when transportation costs are high (low τ) each firm marginal contribution to the location profits is small, whereas when transportation costs are low (high τ) the marginal contribution is large.

Given the linearization in 4.2, the following proposition characterizes the long run geographical equilibrium distribution.

[5] Another way of deriving (4.1) goes back to Thurstone (1927) and presumes that choices are invariant under a uniform expansion of the choice set. If one increases the number of alternatives by adding for each alternative an equal number of identical alternatives, and asks that the probability of choosing a given alternative does not change, the resulting choice process is governed by (4.1).

[6] In the terminology of Bottazzi and Secchi (2007) the intrinsic profit corresponds to the location "intrinsic attractiveness" whereas the marginal profit is the "social externality".

Proposition 4.2. *Consider an economy with two locations, $l = 1, 2$, where Assumptions 1-5 are valid. The economy is populated by N firms, distributed according to $\boldsymbol{n} = (n_1, n_2)$. At the beginning of each period of time a firm is randomly selected, with equal probability over the entire population, to exit the economy. Let $m \in \{1, 2\}$ be the location affected by this exit. After exit takes place, a new firm enters the economy and, conditional on the exit occurred in m, has a probability:*

$$p_l = \frac{a + b(n_l - \delta_{l,m})}{2a + bN}$$

to chose location l, where a and b are given by (4.3)*. This process admits a unique stationary distribution*

$$\pi(\boldsymbol{n}) = \frac{N! C(N, a, b)}{Z(N, a, b)} \prod_{l=1}^{2} \frac{1}{n_l!} \vartheta_{n_l}(a, b)\,, \tag{4.4}$$

where

$$C(N, a, b) = 2a + \left(1 - \frac{1}{N}\right) bN\,, \tag{4.5}$$

$$\vartheta_n(a, b) = \begin{cases} \prod_{h=1}^{n} [a + b(h-1)] & n > 0 \\ 1 & n = 0 \end{cases} \tag{4.6}$$

and $Z(N, a, b)$ is a normalization factor which depends only on the total number of firms N, and the coefficients a, b.

Proof. See Propositions $3.1 - 3.4$ of Bottazzi and Secchi (2007). □

Figure 2 shows results from a simulation of the entry-exit process for two different values of the transportation cost τ. The left panel shows 50000 iterations of the process, whereas the right panel plots the corresponding long run distributions as characterized in Proposition 4.2.

With low transportation costs ($\tau = 0.95$) the long run distribution is clustered around the two extreme values, $x = 0$ and $x = 1$, confirming the prediction of the static analysis. However, the simulation of the entry-exit process (left panel of Fig. 2) shows that agglomeration is only a meta-stable state. One location can become much larger than the other for several time steps, like location 2 which, in the simulation shown, attracts almost all firms in the periods between 2000 and 3500, but at some point the cluster abruptly disappears and the other location can take over. This behavior is well in accordance with the bimodal nature of the equilibrium distribution (c.f. right panel of Fig. 2). In fact, the equilibrium distribution represents the unconditional probability of finding the system in a give state. This probability can thus be very different from the frequency with which this particular state is observed over a finite time window.

Conversely, for higher transportation costs (τ decreases to 0.7), agglomeration is "almost" never observed: firms spatial distribution is now fluctuating around $x^* = 0.5$ (c.f.left panel of Fig. 2). Even if the static analysis predicts agglomeration, the equilibrium distribution of the stochastic system, reported in the right panel of Fig. 2, shows that the most likely geographical distribution has an equal number of firms per location, irrespectively of the fact that the point $x^* = 0.5$ is never a static geographical equilibrium. In general one has the following

Proposition 4.3. *Consider the entry and exit process described in Proposition 4.2. When the marginal profit is bigger than the intrinsic profit, $b > a$, the stationary distribution* (4.4) *is bimodal with modes in $x = 0$ and $x = 1$, when $b < a$ the stationary distribution is unimodal with mode in $x = 0.5$, and when $a = b$ the stationary distribution is uniform.*

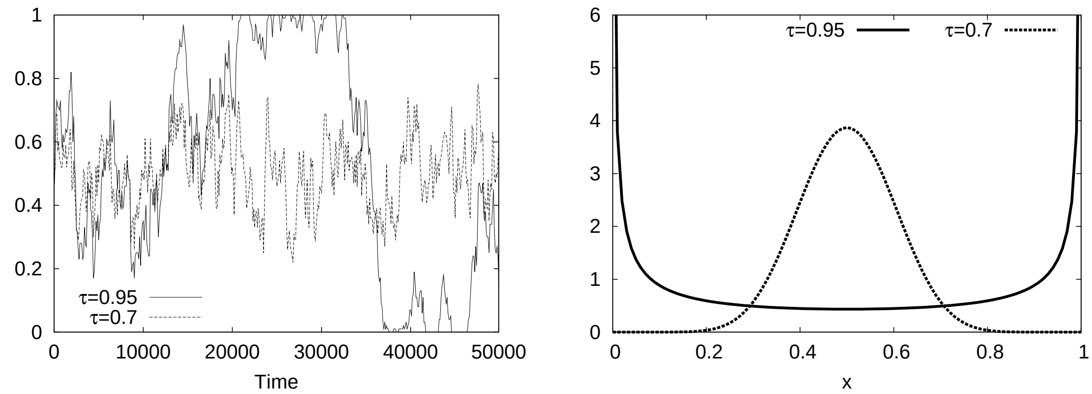

Figure 2: Entry-exit process for different values of the transportation costs. **Left panel**. 50000 simulations of the entry-exit process for different values of the transportation cost τ. **Right panel**. Long run stationary distribution of the entry-exit process simulated in the left panel. In both panels the parameters are $\sigma = 4$, $\alpha = 1$, $I = 400$, $N = 100$.

Proof. See Appendix. □

Given our dynamic locational decision process, the previous proposition clarifies that the shape of the geographical equilibrium distribution does ultimately depend on the relative size of the marginal profit b and the intrinsic profit a. When marginal profits are bigger than intrinsic profits the distribution has mass on the borders of the $[0, 1]$ interval. When marginal profits are lower than intrinsic profits the distribution has higher mass in the middle of this interval, and when they are equal every value of the geographical distribution is as likely.

Rewriting the relation $b \gtrless a$ in Proposition 4.3 using the definitions of a and b in (4.3), it is straightforward to derive the conditions for the unimodality or bimodality of (4.4) in terms of the values of τ, I and α. The left panels of Figs. 3 and 4 have been obtained using these conditions: they show which distributional shape is observed in the different regions of the plane (I, τ) and (α, τ) respectevely. In the white area agglomeration is most likely (bimodal distribution), whereas in the dark area equidistribution is most likely (unimodal distribution). In the right panels the stationary distributions computed at the corresponding points A, B, and C are shown. In both figures these points have been obtained by keeping τ fixed.

The right panel of Fig. 3 shows that moving from small to large values of I while keeping τ fixed, the long run distribution changes from bimodal to unimodal. This is due to the fact that an increase of the number of residents I leads to a decrease of the marginal profit b (c.f. (4.3)). In fact, due to Assumption 5, the more the residents, the more the firms and the smaller the contribution of each firm locational decision to the profits of other firms, that is the smaller the marginal profit. Changing I corresponds to a sort of "size" effect: increasing the size of the economy lowers the externalities so that, due to the entry-exit process, the likelihood to observe agglomeration is lowered.

The right panel of Fig. 3 shows that, keeping τ fixed, an increase in the fixed cost parameter α leads, in general, to more agglomerated economies. This is so because an increase in α decreases the intrinsic profit a while increasing the marginal profit b. More precisely, α determines the scale of the profit differentials. Indeed, on the one hand, the difference between the maximum and the minimum profit is proportional to α and, on the other hand, because of Assumption 5, the higher α the lower N, so that the bigger profit difference is caused by a lower number of firms. As a result, increasing fixed costs decreases the profit each firm

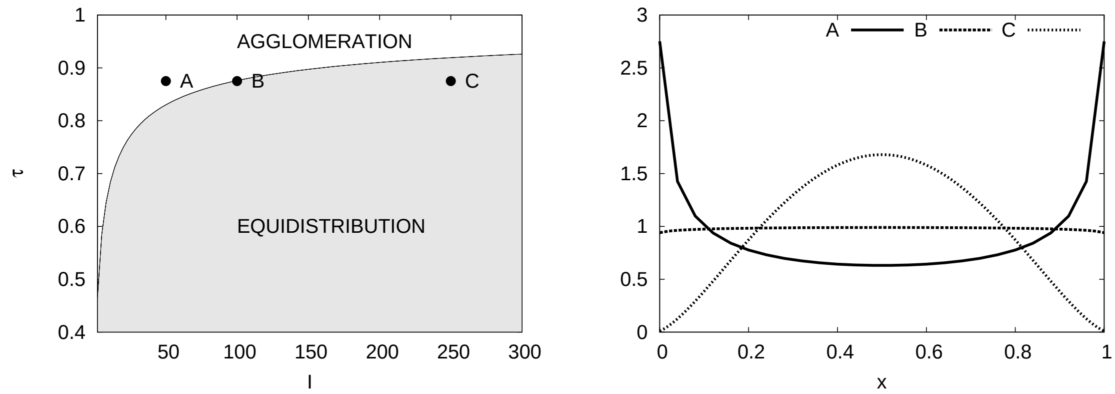

Figure 3: Entry-exit process for different values of the number of residents I. **Left panel**. A portion of the space (I, τ) has been divided into the "agglomeration" area (white) and the "equidistribution" area (shaded) according to Proposition 4.3. **Right panel**. Stationary geographical distributions computed at the points A, B, and C. Other parameters are $\alpha = 1$ and $\sigma = 4$ whereas N is fixed by Ass. 5.

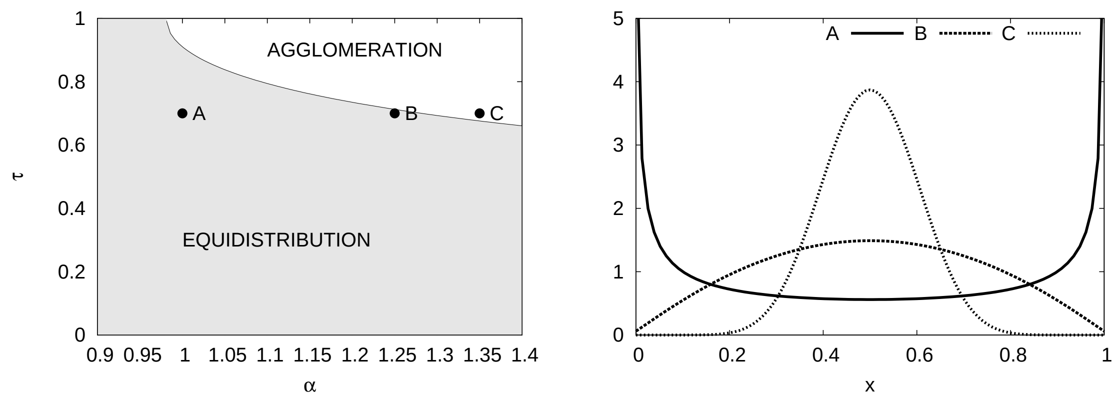

Figure 4: Entry-exit process for different values of the fixed costs α. **Left panel**. A portion of the space (α, τ) has been divided into the "agglomeration" area (white) and the "equidistribution" area (shaded) according to Proposition 4.3. **Right panel**. Stationary geographical distributions computed at the points A, B, and C. Other parameters are $I = 200$ and $\sigma = 4$ whereas N is fixed by Ass. 5.

earns irrespectively of the presence of other firms, and increases the effect of each locational choice to the profits of others. Both effects go in the direction of increasing the likelihood of agglomeration. This is a sort of "scale" effect where increasing the scale of profits increases the likelihood of agglomeration.

Concerning the effect of the transportation cost on the shape of the equilibrium distribution, notice that the expression $\tau^{\sigma-1}/(1+\tau^{\sigma-1})^2$, which appears in (4.3) for both a and b, is an increasing function of τ. Then, increasing the value of this parameter leads to an increase of the marginal profit b and to a decrease of the intrinsic profit a. This means that low transportation costs, that is high values of τ, favor agglomeration, while high transportation costs favor equidistribution. Indeed, when transportation costs are low, the pecuniary externality is relatively weak and the technological externality relatively strong. In terms of the entry-exit process, the choice of a firm to relocate its activity has an high impact on the level of profits. Consequently, it is likely to trigger other relocations and, eventually, a strong agglomeration is observed. Otherwise, when transportation costs are high, the pecuniary and technological externality almost offset each other. This implies that marginal profits are small and intrinsic profits dominate, so that each locational choice has a very small impact on the general level of profits. The attracting force of each location does not depend on the externality term and equidistribution is likely to be observed.

5 Conclusion

We have analyzed a model of firms location in geographical space where firms interacts both indirectly, through market interactions, and directly, through technological externalities, and where workers are not mobile. In this simple framework we have briefly discussed the partial equilibrium static case, identifying the possible geographical equilibria, that is the spatial distributions in which firms do not have any incentive to relocate their activities. We have showed that in this case the "cost sharing" assumption implies long run agglomeration, irrespectively of the number of consumers, of their preferences and of transportation costs. Then, following the discrete choice model presented in Bottazzi and Secchi (2007), we have extended the analysis including heterogeneity in firms preferences and an explicit time dynamics in their choices, and obtained a stochastic model of firms dynamics. We have been able to characterize the long run geographical distribution of the process for different specifications of the economy. This analysis has revealed that, contrary to the static equilibrium analysis, when an explicit entry-exit dynamics is assumed to characterize the locational decision of firms, the economy can evolve towards two different long run scenarios. In the first scenario, where externalities are stronger than intrinsic location profits, which typically occurs for low transportation costs, the long run geographical distribution is bimodal with modes at the borders of the support. Agglomeration is thus the most likely event, but as simulations show, this does not mean that once agglomeration on one side has been achieved, this agglomeration is stable. In fact, turning points exists where the mass of firms moves from one location to the other. In the second scenario the long run geographical distribution has a unique mode. In this case, the most likely event occurs when half of the firms locate in one region and the other half in the other region. However, due to the stochasticity of the process, fluctuations around this level are present. This scenario is typically associated with high transportation costs, and occurs, in general, when the effect of externalities is weak with respect to the locations intrinsic profit.

Summarizing, the main contribution of the foregoing analysis is to show how firms heterogeneity and an individual choice process act as breaks or constraints to firms agglomeration, even when strong incentives to locate in already populous locations exist. Moreover, having introduced an explicit time dimension, we have given history a role. Indeed the time dimension matters in two respects: first, the initial distribution of activities across two locations does influence the subsequent observed distributions and, second, when agglomeration is observed, due to the stochastic fluctuations, it is only a metastable phenomenon. That is, by waiting long enough, the cluster eventually disappears just to be soon recreated, with probability 1/2, in the other location.

Our model can be extended in several directions. First of all, the "cost sharing" assumption, while useful, is admittedly *ad hoc*. A more careful modeling of the determinants and processes of direct, non market mediated, externalities, is probably needed. This generalization could encompass not only positive externalities, as the one analyzed in this chapter and inspired at the notion of technological and/or knowledge spillover, but also negative ones, as due to pollution and/or congestion effects. A second possible extension of the model would be to generalize consumers behavior along the same lines we have taken to describe firms behavior. Whereas in the present version of the model consumers are homogenous and maximize the same CES utility function, it would be interesting to assume that consumers are heterogeneous and to explicitly model their consumption decision in time. In that case, changing the size of the economy would imply, due to varying idiosyncrasies in consumers demand, a change in the amplitude of profits fluctuations. This, in turn, might also impact the likelihood of agglomeration, and possibly change the overall picture.

In any case, we are aware that the ultimate test bed will be to confront our model with real data. An interesting aspect of the discrete choice model we implemented is that it can be quite easily brought to the data. An exercise in this direction has already been performed in Bottazzi et al. (2008) where the parameters characterizing the geographical equilibrium distribution have been estimated in several sectors of the Italian manufacturing industry. The present work moves in the direction of developing a theoretical framework able to provide deeper and more informative economic interpretations of these econometric exercises.

A Appendix

Proof of Proposition 2.1 and Corollary 2.1 Average profits follow in a straightforward way by computing $\bar{\pi} = x\pi_1(x) + (1-x)\pi_2(x)$ with $\pi_1(x)$ and $\pi_2(x)$ as in (2.12) and in (2.17). Total costs in each location are equal because (2.14) implies:

$$\sum_{i=1,n_1} \alpha_i = \sum_{j=1,n_2} \alpha_j = \frac{N\alpha}{2} .$$

□

Proof of Proposition 3.1 First, we show that the unique firms distribution where profits are equal is $x = 0.5$. Take profits as given by (2.17). Setting $\pi_1(x) = \pi_2(x)$ one finds a first order equation in x whose only solution is $x = 0.5$.

Regarding geographical equilibria first notice that they are the Pure Strategy Nash Equilibria (PSNE) of the one stage game where each firm in a group of N firms (N even), has to choose to be located in $l = 1$ or $l = 2$ and profits are given by (2.17). Denote firm $i = 1, \ldots, N$ strategy as s_i. Each firm i can choose to be located either in 1, $s_i = 1$, or in 2, $s_i = 0$. The strategy space has thus 2^N elements, and each strategy profile will be denoted as s. With s_{-i} we indicate instead the strategy profile of $N-1$ firms but firm i. Define also

$$x(s) = \frac{\sum_{i=1,\ldots,N} s_i}{N} .$$

To complete the formalization of the game we have to specify each firm payoff for any strategy profile s. When $s_i = 1$, firm i payoff π_i is given by

$$\pi_i(1, s_{-i}) \equiv \pi_1(x(s)) ,$$

where $\pi_1(x)$ is taken from (2.17) and $x(s)$ is defined above. When instead $s_i = 0$, firm i payoff is

$$\pi_i(0, s_{-i}) \equiv \pi_2(x(s)) ,$$

where $\pi_2(x)$ is taken from (2.17) as well. For example if all firms are choosing location $l = 1$, so that $x = 1$, it holds $\pi_i = \pi_1(1)$ for all $i = 1, \ldots, N$. If half of the firms are located in $l = 1$ and the other half in $l = 2$, so that $x = 0.5$, we have $\pi_i = \pi_1(0.5)$ when $s_i = 1$ and $\pi_i = \pi_2(0.5)$ otherwise. A strategy profile s^* is a PSNE if and only if

$$\pi_i(s^*) \geq \pi_i(s_i, s^*_{-i}) \quad \text{for} \quad s_i = 0, 1, \text{and for all} \quad i = 1, \ldots, N . \tag{A.1}$$

When s^* is an interior strategy profile, (A.1) can be rewritten using profits π_1 and π_2:

$$\pi_1(x(s^*)) \geq \pi_2 \left(x(s^*) - \frac{1}{N} \right) \tag{A.2}$$

$$\pi_2(x(s^*)) \geq \pi_1 \left(x(s^*) + \frac{1}{N} \right) \tag{A.3}$$

A border strategy profile s^*, that is where all players are choosing l, is instead a PSNE when only the l^{th} condition above is satisfied. We start by showing that every s^* such that $x(s^*) = 1$ or $x(s^*) = 0$ is a PSNE profile. From (2.17) it holds that $\pi_1(1) > \pi_2(x)$ for all $x \in [0, 1)$ and

$\pi_2(0) > \pi_1(x)$ for all $x \in (0, 1]$, so that (A.2) and (A.3) are, respectively, satisfied. We conclude the proof by showing that every strategy profile s^* with $x(s^*) = x^* = 0.5$, that the only other candidate to be geographical equilibria, is not an PSNE. To be a PSNE, in terms of conditions (A.2) and (A.3), we have to show that

$$\pi_1(0.5) \geq \pi_2\left(0.5 - \frac{1}{N}\right) \forall i = 1, \ldots, N\,, \tag{A.4}$$

$$\pi_2(0.5) \geq \pi_1\left(0.5 + \frac{1}{N}\right) \forall i = 1, \ldots, N\,. \tag{A.5}$$

Since our locations are identical and x is the concentration of firms who choose location 1, by symmetry it holds that $\pi_1(x) = \pi_2(1-x)$. Using this relation we can rewrite (A.4-A.5) as:

$$\pi_1(0.5) \geq \pi_1\left(0.5 + \frac{1}{N}\right) \forall i = 1, \ldots, N\,,$$

$$\pi_1(0.5) \geq \pi_1\left(0.5 - \frac{1}{N}\right) \forall i = 1, \ldots, N\,,$$

which are always satisfied as long as the function $\pi_1(x)$ is decreasing at $x = 0.5$. Computing $d\pi_1(x)/dx|_{x=0.5}$ shows that this is the never the case, which concludes the proof (see the following proofs for the explicit expression $d\pi_1(x)/dx|_{x=0.5}$). □

Proof of Proposition 4.2 In order to linearize the exponential of profits around x^*, consider first the Taylor expansion up to the first order of each term in (2.17) as a function of $z = x - 0.5$, that is,

$$\pi_l(z) = \pi_l(0.5) + z\left.\frac{d\pi_l(x)}{dx}\right|_{x=0.5} + o(z^2)\,,$$

where the derivative is given by

$$\left.\frac{d\pi_l(x)}{dx}\right|_{x=0.5} = (-)^{(l-1)}\left(-\frac{4I}{N\sigma}\left(\frac{1-t^{\sigma-1}}{1+t^{\sigma-1}}\right)^2 + 2\alpha\right) \quad l = 1, 2\,.$$

It follows, using the properties of the exponential function, that the linearization of e^{π_l} around z is given by

$$e^{\pi_l(z)} = e^{\pi_l(0.5)}\left(1 + z\left.\frac{d\pi_l(x)}{dx}\right|_{x=0.5}\right) + o(z^2)\,. \tag{A.6}$$

Notice that due to the form of (4.1), multiplicative factors, in our case where $\exp(\pi_l(0.5))$, can be factorized. Doing so and rewriting (A.6) as a function of the number of firms n_l, $l = 1, 2$, we obtain the expression of the linearized exponential payoff c_l,

$$c_1 = 1 - \frac{2}{N\sigma}\left(2I\frac{(1-t^{\sigma-1})^2}{(1+t^{\sigma-1})^2} - N\alpha\sigma\right)\left(\frac{n_1}{N} - \frac{1}{2}\right)\,,$$

$$c_2 = 1 + \frac{2}{N\sigma}\left(2I\frac{(1-t^{\sigma-1})^2}{(1+t^{\sigma-1})^2} - N\alpha\sigma\right)\left(\frac{1}{2} - \frac{n_2}{N}\right)\,.$$

This shows that a and b are given by

$$\begin{aligned} a &= 1 + \frac{1}{N\sigma}\left(2I\frac{(1-t^{\sigma-1})^2}{(1+t^{\sigma-1})^2} - N\alpha\sigma\right), \\ b &= -\frac{1}{N}\frac{2}{N\sigma}\left(2I\frac{(1-t^{\sigma-1})^2}{(1+t^{\sigma-1})^2} - N\alpha\sigma\right), \end{aligned}$$

which, using Ass. 5 to eliminate the dependence on N, correspond to (4.3). □

Proof of Proposition 4.3 We shall use Lemma A.1 (proved below) which states that the density (4.4) is uniform when $a = b$, is bimodal with modes in 0 and N when $b > a$, and has a unique mode in $N/2$ when $b < a$. This together with the expressions of a and b in (4.3) proves the proposition. In terms of the models parameters $b \gtrless a$ is equivalent to

$$\frac{4\alpha^2\sigma\tau^{\sigma-1}}{I(1+\tau^{\sigma-1})^2} \gtrless 1 - \frac{4\alpha\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2},$$

which, taking all the terms in τ on the left hand side, can be easily rewritten as

$$4\alpha\left(1+\frac{\alpha\sigma}{I}\right) \gtrless \frac{(1+\tau^{\sigma-1})^2}{\tau^{\sigma-1}}.$$

□

Lemma A.1. *Consider the distribution with support on the integers between* 0 *and* N *(even) and with probability density* $\pi(n)$ *as defined in* (4.4)*. When* $a = b$ *the distribution is uniform, that is,* $\pi(n) = \pi(m)$ *for every* $n, m = 0, \ldots, N$*. When* $b > a$ *the distribution is bimodal with modes in* 0 *and* N*, that is,* $\pi(0) = \pi(N) > \pi(n)$ *for every* $n \neq 0, N$*. When* $b < a$ *the distribution i unimodal with mode in* $N/2$*, that is,* $\pi(N/2) > \pi(n)$ *for every* $n \neq N/2$*.*

Proof of Lemma A.1 First notice that from (4.4) it follows that the distribution is symmetric around $N/2$, that is $\pi(N/2+n) = \pi(N/2-n)$ for every $n = 0, \ldots, N/2$. This property allows to consider only half of the support, namely the set $\{0, \ldots, N/2\}$.

When $a = b$, $\theta_n(a,b)$ reduces to $n!$. As a result the probability density becomes

$$\pi(n) = \frac{N!C(N,a,b)}{Z(N,a,b)} \quad \forall n,$$

so that the distribution is uniform. The rest of the lemma will be proved by induction. First consider

$$\begin{aligned} \pi(0) &\gtrless \pi(1) \\ \frac{N!C(N,a,b)}{Z(N,a,b)}\frac{1}{N!}\theta_N(a,b) &\gtrless \frac{N!C(N,a,b)}{Z(N,a,b)}\frac{1}{(N-1)!}\theta_1(a,b)\theta_{N-1}(a,b) \\ a(a+b)\ldots(a+b(N-1)) &\gtrless Na^2(a+b)\ldots(a+b(N-2)) \\ b &\gtrless a. \end{aligned} \tag{A.7}$$

Second consider

$$
\begin{aligned}
\pi(n) &\gtrless \pi(n+1) \\
\frac{N!C(N,a,b)}{Z(N,a,b)}\frac{1}{n!(N-n)!}\theta_n(a,b)\theta_{N-n}(a,b) &\gtrless \frac{N!C(N,a,b)}{Z(N,a,b)}\frac{1}{(n+1)!(N-n-1)!}\theta_{n+1}(a,b)\theta_{N-n-1}(a,b) \\
(n+1)a(a+b)\ldots(a+b(n-1))a(a+b)\ldots(a+b(N-n-1)) &\gtrless \\
\gtrless (n+1)a(a+b)\ldots(a+bn)a(a+b)\ldots(a+b(N-n-2)) \\
b(N-2n-1) &\gtrless B(N-2n-1) \\
b &\gtrless a\,, \qquad \text{(A.8)}
\end{aligned}
$$

where the last step requires $n \leq N/2-1$, which is our case. From A.7 and A.8 and reasoning by induction, it follows that when $b > a$ the maximum is in $\pi(0)$ (and by symmetry also in $\pi(N)$), whereas, when $b < a$, the maximum is in $\pi(N/2)$. □

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